This paper works with the chain rule.

Hope I can help you with learning quantum mechanics.

Case one: x and y are functions of one variable u

We use differentiable functions:

$$z = f(x,y), x = g(u), y = h(u)$$

z is a function of two variables x and y.

x and y are functions of one variable u.

We rewrite z:

$$z = f\big(g(u), h(u)\big)$$

We derive:

$$\frac{d}{du}z = \frac{\partial}{\partial x}z \cdot \frac{d}{du}x + \frac{\partial}{\partial y}z \cdot \frac{d}{du}y$$

We see that deriving z with respect to u involves four (partial) derivatives:

$$\frac{\partial}{\partial x}z, \qquad \frac{\partial}{\partial y}z, \qquad \frac{d}{du}x, \qquad \frac{d}{du}y$$

Example

$$z = f(x,y) = 4 \cdot x^2 + 3 \cdot y^2$$

$$x = x(u) = u^2 + 2 \cdot u + 3$$

$$y = y(u) = e^{3 \cdot u^2}$$

We calculate two partial derivatives:

$$\frac{\partial}{\partial x}z = \frac{\partial}{\partial x}f(x,y) = (4 \cdot x^2 + 3 \cdot y^2) = 8 \cdot x$$

$$\frac{\partial}{\partial y}z = \frac{\partial}{\partial y}f(x,y) = (4 \cdot x^2 + 3 \cdot y^2) = 6 \cdot y$$

We calculate two derivatives:

$$\frac{d}{du}x = \frac{d}{du}x(u) = 2 \cdot u + 2$$

$$\frac{d}{du}y = \frac{d}{du}y(u) = 6 \cdot u \cdot e^{3 \cdot u^2}$$

We insert:

$$\frac{d}{du}z = \frac{\partial}{\partial x}z \cdot \frac{d}{du}x + \frac{\partial}{\partial y}z \cdot \frac{d}{du}y =$$

$$(8 \cdot x) \cdot (2 \cdot u + 2) + (6 \cdot y) \cdot \left(6 \cdot u \cdot e^{3 \cdot u^2}\right) =$$

$$16 \cdot x \cdot u + 16 \cdot x + 36 \cdot u \cdot y \cdot e^{3 \cdot u^2} =$$

$$16 \cdot x \cdot (u + 1) + 36 \cdot u \cdot y \cdot e^{3 \cdot u^2} =;$$

We replace variables x and y by their definitions:

$$16 \cdot x \cdot (u+1) + 36 \cdot u \cdot y \cdot e^{3 \cdot u^2} \rightarrow$$

$$16 \cdot (u^2 + 2 \cdot u + 3) \cdot (u+1) + 36 \cdot u \cdot \left(e^{3 \cdot u^2}\right) \cdot e^{3 \cdot u^2} =$$

$$16 \cdot (u^3 + u^2 + 2 \cdot u^2 + 2 \cdot u + 3 \cdot u + 3) + 36 \cdot u \cdot e^{6 \cdot u^2} =$$

$$16 \cdot (u^3 + 3 \cdot u^2 + 5 \cdot u + 3) + 36 \cdot u \cdot e^{6 \cdot u^2} =$$

$$16 \cdot u^3 + 48 \cdot u^2 + 80 \cdot u + 48 + 36 \cdot u \cdot e^{6 \cdot u^2}$$

We check this. Instead of building the partial derivatives according to the scheme above we resolve $z = f(x,y) = 4 \cdot x^2 + 3 \cdot y^2$ by substituting $x = x(u) = u^2 + 2 \cdot u + 3$ and $y = y(u) = e^{3 \cdot u^2}$:

$$z = 4 \cdot x^2 + 3 \cdot y^2 = 4 \cdot (u^2 + 2 \cdot u + 3)^2 + 3 \cdot \left(e^{3 \cdot u^2}\right)^2 =$$

$$4 \cdot (u^2 + 2 \cdot u + 3) \cdot (u^2 + 2 \cdot u + 3) + 3 \cdot e^{6 \cdot u^2} =$$

$$4 \cdot (u^4 + 2 \cdot u^3 + 3 \cdot u^2 + 2 \cdot u^3 + 4 \cdot u^2 + 6 \cdot u + 3 \cdot u^2 + 6 \cdot u + 9) + 3 \cdot e^{6 \cdot u^2} =$$

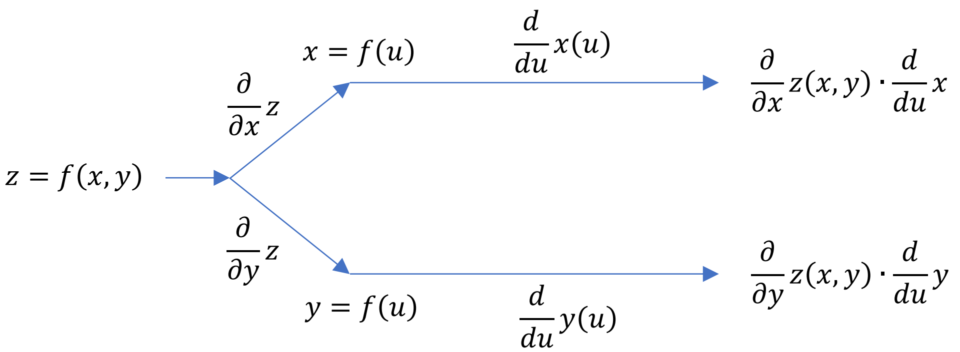
$$4 \cdot (u^4 + 4 \cdot u^3 + 10 \cdot u^2 + 12 \cdot u + 9) + 3 \cdot e^{6 \cdot u^2} =$$

$$4 \cdot u^4 + 16 \cdot u^3 + 40 \cdot u^2 + 48 \cdot u + 36 + 3 \cdot e^{6 \cdot u^2}$$

We differentiate with respect to u:

$$\frac{d}{du} z = 16 \cdot u^3 + 48 \cdot u^2 + 80 \cdot u + 48 + 36 \cdot u \cdot e^{6 \cdot u^2}$$

This often is presented as a tree diagram:



In summa we get:

$$\frac{\partial}{\partial u} z(x,y) = \frac{\partial}{\partial x} z(x,y) \cdot \frac{d}{du} x(u) + \frac{\partial}{\partial y} z(x,y) \cdot \frac{d}{du} y(u)$$

Note: We can write $\frac{d}{du} x(u)$ or $\frac{\partial}{\partial u} x(u)$ because x depends of u only.

Case two: x and y are functions of two variables u and v

We use differentiable functions:

$$z = f(x, y), x = g(u, v), y = h(u, v)$$

z is a function of two variables x and y.

x and y are functions of two variables u and v.

We rewrite z:

$$z = f\big(g(u, v), h(u, v)\big)$$

We derive:

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial u}$$

$$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial v} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial v}$$

Note: in contrast to the case of one variable we now have partial derivatives only.

We see that deriving z with respect to u involves six partial derivatives:

$$\frac{\partial z}{\partial x}, \quad \frac{\partial z}{\partial y}, \quad \frac{\partial x}{\partial u}, \quad \frac{\partial x}{\partial v}, \quad \frac{\partial y}{\partial u}, \quad \frac{\partial y}{\partial v}$$

Example

$$z = f(x, y) = 4 \cdot x^2 + 3 \cdot y^2$$

$$x = x(u, v) = 5 \cdot u^2 + 7 \cdot v^3$$

$$y = y(u, v) = e^{3u+5v}$$

We calculate two partial derivatives:

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x} f(x, y) = (4 \cdot x^2 + 3 \cdot y^2) = 8 \cdot x$$

$$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y} f(x, y) = (4 \cdot x^2 + 3 \cdot y^2) = 6 \cdot y$$

Note: these are the same as in the example above.

We calculate the rest:

$$\frac{\partial x}{\partial u} = 10 \cdot u$$

$$\frac{\partial x}{\partial v} = 21 \cdot v^2$$

$$\frac{\partial y}{\partial u} = 3 \cdot e^{3u+5v}$$

$$\frac{\partial y}{\partial v} = 5 \cdot e^{3u+5v}$$

We calculate $\frac{\partial z}{\partial u}$:	We calculate $\frac{\partial z}{\partial v}$:
$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial u} =$ $8 \cdot x \cdot 10 \cdot u + 6 \cdot y \cdot 3 \cdot e^{3u+5v} =$ $80 \cdot x \cdot u + 18 \cdot y \cdot e^{3u+5v}$	$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial v} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial v} =$ $8 \cdot x \cdot 21 \cdot v^2 + 6 \cdot y \cdot 5 \cdot e^{3u+5v} =$ $168 \cdot x \cdot v^2 + 30 \cdot y \cdot e^{3u+5v}$

We replace variables x and y by their definitions:

$\frac{\partial z}{\partial u} = 80 \cdot x \cdot u + 18 \cdot y \cdot e^{3u+5v} \rightarrow$ $\frac{\partial z}{\partial u} = 80 \cdot (5 \cdot u^2 + 7 \cdot v^3) \cdot u + 18 \cdot (e^{3u+5v}) \cdot e^{3u+5v} =$ $400 \cdot u^3 + 560 \cdot u \cdot v^3 + 18 \cdot e^{6u+10v}$	$\frac{\partial z}{\partial v} = 168 \cdot x \cdot v^2 + 30 \cdot y \cdot e^{3u+5v} \rightarrow$ $\frac{\partial z}{\partial v} = 168 \cdot (5 \cdot u^2 + 7 \cdot v^3) \cdot v^2 + 30 \cdot (e^{3u+5v}) \cdot e^{3u+5v} =$ $840 \cdot u^2 \cdot v^2 + 1176 \cdot v^5 + 30 \cdot e^{6u+10v}$

We check this. Instead of building the partial derivatives we resolve $z = f(x, y) = 4 \cdot x^2 + 3 \cdot y^2$ by substituting $x = x(u, v) = 5 \cdot u^2 + 7 \cdot v^3$ and $y = y(u, v) = e^{3u+5v}$:

$$z = 4 \cdot x^2 + 3 \cdot y^2 = 4 \cdot (5 \cdot u^2 + 7 \cdot v^3)^2 + 3 \cdot (e^{3u+5v})^2 =$$

$$4 \cdot (5 \cdot u^2 + 7 \cdot v^3) \cdot (5 \cdot u^2 + 7 \cdot v^3) + 3 \cdot e^{6u+10v} =$$

$$4 \cdot (25 \cdot u^4 + 70 \cdot u^2 \cdot v^3 + 49 \cdot v^6) + 3 \cdot e^{6u+10v} =$$

$$100 \cdot u^4 + 280 \cdot u^2 \cdot v^3 + 196 \cdot v^6 + 3 \cdot e^{6u+10v}$$

We build $\frac{\partial z}{\partial u}$:	We build $\frac{\partial z}{\partial v}$:
$\frac{\partial z}{\partial u} = \frac{\partial}{\partial u}(100 \cdot u^4 + 280 \cdot u^2 \cdot v^3 + 196 \cdot v^6 + 3 \cdot e^{6u+10v}) =$ $400 \cdot u^3 + 560 \cdot u \cdot v^3 + 18 \cdot e^{6u+10v}$	$\frac{\partial z}{\partial v} = \frac{\partial}{\partial v}(100 \cdot u^4 + 280 \cdot u^2 \cdot v^3 + 196 \cdot v^6 + 3 \cdot e^{6u+10v}) =$ $840 \cdot u^2 \cdot v^2 + 1176 \cdot v^5 + 30 \cdot e^{6u+10v}$