Wave functions can be written as vectors or continuous functions.

This paper takes the discrete probability of a dice and transforms it to a continuous probability function.

A similar example with a set of sampled data you find in:

Lin, M. (2018) Techniques of Discrete Function Transfers into Continuous Function in Practice. Engineering, 10, 680-687.

Accessible via the link:

https://www.scirp.org/journal/PaperInformation.aspx?PaperID=87875

Hope I can help you with learning quantum mechanics.

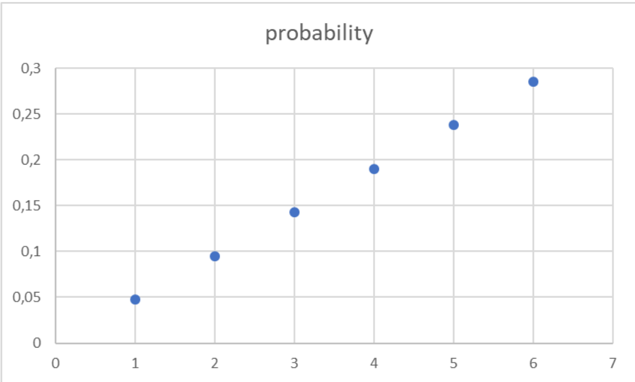
We take a dice with six sides, an unfair one. The chances are not uniform distributed, they are:

Face	1	2	3	4	5	6
Probability	$\frac{1}{21}$	$\frac{2}{21}$	$\frac{3}{21}$	$\frac{4}{21}$	$\frac{5}{21}$	$\frac{6}{21}$

The sum of all probabilities must give 1:

$$\sum_{i=1}^{6} p_i = \frac{1+2+3+4+5+6}{21} = 1$$

The probabilities:



This dice has six degrees of freedom, the phase space has six dimensions.

We can use six orthogonal vectors:

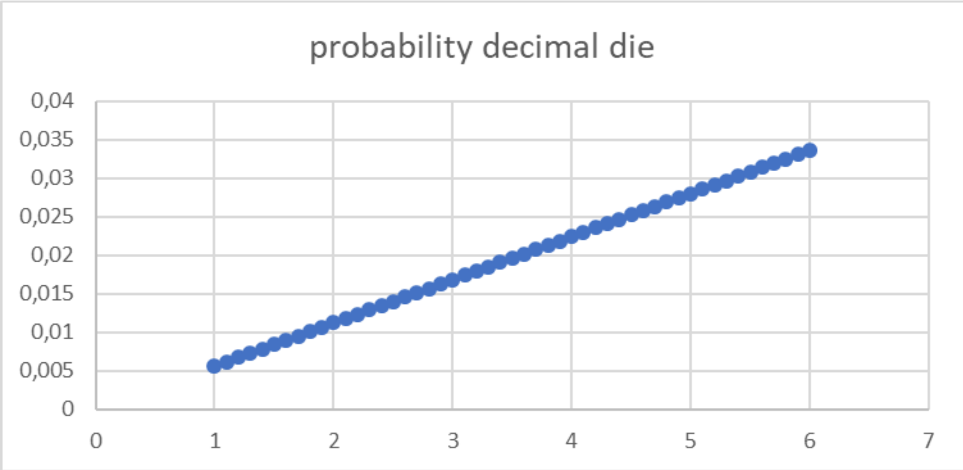
$$d_1 = \frac{1}{21}\begin{pmatrix}1\\0\\0\\0\\0\\0\end{pmatrix}, \frac{2}{21}\begin{pmatrix}0\\1\\0\\0\\0\\0\end{pmatrix}, \frac{3}{21}\begin{pmatrix}0\\0\\1\\0\\0\\0\end{pmatrix}, \frac{4}{21}\begin{pmatrix}0\\0\\0\\1\\0\\0\end{pmatrix}, \frac{5}{21}\begin{pmatrix}0\\0\\0\\0\\1\\0\end{pmatrix}, \frac{6}{21}\begin{pmatrix}0\\0\\0\\0\\0\\1\end{pmatrix}$$

The wave function for this dice in the phase space:

$$|\psi\rangle = \begin{pmatrix}1/21\\2/21\\3/21\\4/21\\5/21\\6/21\end{pmatrix}$$

Next, we give that dice more faces 1.1, 1.2, … , 5.9, 6.0.

Again, the dice is unfair. The probabilities:



We have a phase space of 51 dimensions.

$$d_1 = \frac{1}{178.5}\begin{pmatrix}1\\ \cdot\\ \cdot\\ \cdot\\ \cdot\\ \cdot\end{pmatrix}, \frac{2}{178.5}\begin{pmatrix}0\\ 1\\ \cdot\\ \cdot\\ \cdot\\ \cdot\end{pmatrix}, \dots, \frac{60}{178.5}\begin{pmatrix}\cdot\\ \cdot\\ \cdot\\ \cdot\\ \cdot\\ 1\end{pmatrix}$$

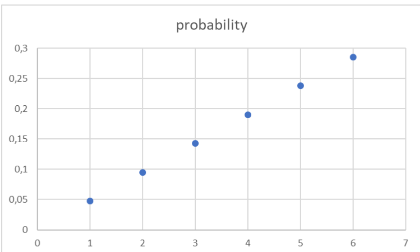
The wave function for this dice in the phase space:

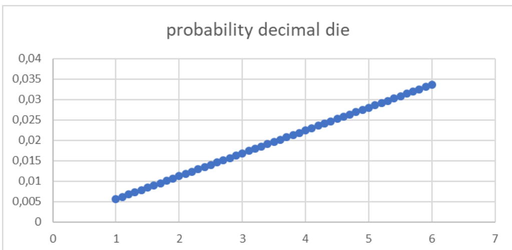
$$|\psi\rangle = \begin{pmatrix}{}^{1}/_{178.5}\\ \cdot\\ \cdot\\ \cdot\\ \cdot\\ {}^{60}/_{178.5}\end{pmatrix}$$

Obviously, this is not easy to handle.

We need another approach.

If we compare the pictures of the single probabilities, we see that the ratio is preserved, the probabilities become smaller and are approaching zero:





The cumulated probabilities always give 1:

$$\sum_{i=1}^{n} p_i = 1$$

For the continuous case we replace the discrete probabilities by a probability density $\tilde{p}(x)$. Integration over $\tilde{p}(x)$ must give 1:

$$\int_{-\infty}^{\infty} \tilde{p}(x)dx = 1$$

Because the probability outside the “faces” is zero the limits of the integral reduces to:

$$\int_{1}^{6} \tilde{p}(x)dx = 1$$

Our probability density is not distributed uniform but raises from "1" to the "6" by a factor of 6:

$$\frac{\tilde{p}(6)}{\tilde{p}(1)} = 6$$

We assume linearity and get:

$$\tilde{p}(x) = \frac{6}{c}x$$

Note: the constant c is necessary for normalizing the integral to 1.

The probability to get any value between 1 and 6 is 1.

$$\int_{1}^{6} \tilde{p}(x)dx = \int_{1}^{6} \frac{6x}{c}dx = \frac{6}{c}\int_{1}^{6} xdx =$$

$$\frac{6}{c}\left[\frac{x^2}{2}\right]_1^6 = \frac{6}{2c}[x^2]_1^6 = \frac{6}{2c}(36-1) = \frac{105}{c} = 1$$

$$c = 105$$

We get the probability density:

$$\tilde{p}(x) = \frac{6}{105}x = \frac{2}{35}x$$

We check:

$$\int_{1}^{6} \tilde{p}(x)dx = 1$$

$$\int_{1}^{6} \frac{2}{35}xdx = \frac{2}{35}\int_{1}^{6} xdx = \frac{2}{70}[x^2]_1^6 =$$

$$\frac{2}{70}(36-1) = 1$$

Summarizing

The discrete probabilities for the faces 1 to 6, the values of the discrete wave function were:

$$\frac{1}{21},\frac{2}{21},\frac{3}{21},\frac{4}{21},\frac{5}{21},\frac{6}{21}$$

The probability density (the wave function) we got out of the discrete case:

$$\psi(x)=\frac{2}{35}x$$

In the discrete case we have a probability for the “side 1” of the die, the x-value 1 of $\frac{1}{21}$.

In the continuous case we have a probability for the x-value 1 of zero and for any single point on the x-axis.

What we can do is to calculate probabilities for intervals on the x-axis.

Check: We compare the probabilities for “the upper half” of the die, the values from 4 to 6.

Discrete 6-sided:

$$p(4{,}5{,}6)=\frac{4+5+6}{21}=\frac{5}{7}\sim 0.7143$$

Discrete 51-sided:

$$p(3.5,3.6,\dots\ 5.9,6.0)=\frac{\sum_{i=3.5}^{6.0} i}{178{,}5}=\frac{123.5}{178{,}5}\sim 0.6919$$

Continuous:

$$p(3.5-6)=\frac{2}{35}\int_{3.5}^{6} x dx=$$

$$\frac{2}{70}[x^2]_{3.5}^{6}=\frac{2}{70}(36-12.25)=\frac{47.5}{70}$$

$$=\frac{19}{28}\sim 0.6786$$

The probabilities from the discrete case are converging to the probability of the continuous case.