If we want to integrate expressions with sin or cos, it may be helpful to replace them by their complex counterparts $sin(x) = \frac{e^{ix}-e^{-ix}}{2\cdot i}$ and $cos(x) = \frac{e^{ix}+e^{-ix}}{2}$, $tan(x) = \frac{i}{e^{2ix}+1} - \frac{i}{e^{-2ix}+1}$.

We try this with three examples:

$$\int e^x \cdot cos(x)dx$$

$$\int cos(x) \cdot sin(x)dx$$

$$\int e^{2x} \cdot cos(3x)dx$$

More information you may find at:

https://en.wikibooks.org/wiki/Calculus/Integration_techniques/Integration_by_Complexifying

Hope I can help you with learning quantum mechanics.

Complexifying is a method to solve exponentials containing trigonometric functions.

We want to solve the first example:

$$\int e^x \cdot cos(x)dx$$

We know:

$$cos(x) = Re(e^{ix})$$

Note: $Re(e^{ix})$ is the real part of the complex e^{ix}.

We have:

$$\int e^x \cdot cos(x)dx = Re\left(\int e^x \cdot e^{ix}dx\right) = Re\left(\int e^{x(1+i)}dx\right)$$

For better readability we omit the "Re" and solve:

$$\int e^{x\cdot(1+i)}dx$$

We get:

$$\int e^{x\cdot(1+i)}dx = \frac{e^{x\cdot(1+i)}}{(1+i)} = \frac{e^x \cdot e^{ix}}{(1+i)}$$

We need to extract the Real part:

$$\frac{e^x \cdot e^{ix}}{(1+i)} = \frac{(1-i)\cdot e^x \cdot e^{ix}}{(1+i)\cdot(1-i)} =$$

$$\frac{1}{2}\cdot\left(e^x \cdot e^{ix} - i \cdot e^x \cdot e^{ix}\right) =$$

$$\frac{1}{2}\cdot e^x \cdot \left(e^{ix} - i \cdot e^{ix}\right)$$

We transform in $\sin/cos$:

$$\frac{1}{2}\cdot e^x \cdot \Big(cos(x) + i \cdot sin(x) - i \cdot \big(cos(x) + i \cdot sin(x)\big)\Big) =$$

$$\frac{1}{2}\cdot e^x \cdot \big(cos(x) + i \cdot sin(x) - i \cdot cos(x) + sin(x)\big)$$

We extract the real part only:

$$\frac{1}{2}\cdot e^x \cdot \big(cos(x) + sin(x)\big)$$

We get the solution:

$$\int e^x \cdot cos(x)dx = \frac{1}{2}\cdot e^x \cdot \big(cos(x) + sin(x)\big)$$

We check with the product rule:

$$\frac{d}{dx}\left(\frac{1}{2}\cdot e^x \cdot \big(cos(x) + sin(x)\big)\right) =$$

$$\frac{1}{2}\cdot e^x\cdot\big(cos(x)+sin(x)\big)+\frac{1}{2}\cdot e^x\cdot\big(-sin(x)+cos(x)\big)=$$

$$\frac{1}{2}\cdot e^x\cdot\big(cos(x)+sin(x)-sin(x)+cos(x)\big)=$$

$$e^x\cdot cos(x)$$

The same example, working with exponentials:

$$\int e^x\cdot cos(x)dx$$

We know:

$$cos(x)=\frac{e^{ix}+e^{-ix}}{2}$$

We insert:

$$\int e^x\cdot\frac{e^{ix}+e^{-ix}}{2}dx=$$

$$\frac{1}{2}\int e^x\cdot e^{ix}+e^x\cdot e^{-ix}dx=$$

$$\frac{1}{2}\left(\int e^{x(1+i)}dx+\int e^{x(1-i)}dx\right)=$$

$$\frac{1}{2}\left(\frac{e^{x(1+i)}}{1+i}+\frac{e^{x(1-i)}}{1-i}\right)=$$

$$\frac{1}{2}\left(\frac{(1-i)e^{x(1+i)}+(1+i)e^{x(1-i)}}{(1+i)(1-i)}\right)=$$

$$\frac{1}{4}\big((1-i)e^{x(1+i)}+(1+i)e^{x(1-i)}\big)=$$

$$\frac{1}{4}\Big(e^x\left((1-i)e^{ix}+(1+i)e^{-ix}\right)\Big)=$$

$$\frac{1}{4}e^x\left(e^{ix}+e^{-ix}-i\big(e^{ix}-e^{-ix}\big)\right)=$$

$$\frac{1}{4}e^x\left(2cos(x)-i\big(2isin(x)\big)\right)=$$

$$\frac{1}{2}e^x\big(cos(x)+sin(x)\big)$$

This is the same solution as above.

Example two:

$$\int cos(x)sin(x)dx$$

We replace cos and sin by the respective exponential expressions:

$$\int \frac{e^{ix}+e^{-ix}}{2}\cdot\frac{e^{ix}-e^{-ix}}{2\cdot i}dx =$$

$$-\frac{1}{4i}\int \left(e^{ix}+e^{-ix}\right)\cdot\left(e^{ix}-e^{-ix}\right)dx =$$

$$-\frac{1}{4i}\int e^{2ix}-e^{ix}e^{-ix}+e^{ix}e^{-ix}-e^{-2ix}dx =$$

$$-\frac{1}{4i}\int e^{2ix}-e^{-2ix}dx =$$

$$-\frac{1}{4i}\int 2isin(2x)dx =$$

$$-\frac{1}{2}\int sin(2x)dx =$$

$$-\frac{1}{4}cos(2x)$$

Example three:

We want to solve:

$$\int e^{2x}\cdot cos(3x)dx$$

We know:

$$cos(3x)=\frac{e^{i3x}+e^{-i3x}}{2}$$

We insert:

$$\int e^{2x}\cdot\frac{e^{i3x}+e^{-i3x}}{2}dx =$$

$$\frac{1}{2}\int e^{2x}\cdot e^{i3x}+e^{2x}\cdot e^{-i3x}dx =$$

$$\frac{1}{2}\left(\int e^{x(2+i3)}+e^{x(2-i3)}dx\right) =$$

$$\frac{1}{2}\left(\frac{e^{x(2+i3)}}{2+i3}+\frac{e^{x(2-i3)}}{2-i3}\right) =$$

$$\frac{1}{2}\left(\frac{(2-i3)\left(e^{x(2+i3)}\right)+(2+i3)\left(e^{x(2-i3)}\right)}{(2+i3)(2-i3)}\right) =$$

$$\frac{1}{26}\left((2-i3)e^{x(2+i3)}+(2+i3)e^{x(2-i3)}\right) =$$

$$\frac{1}{26}\left(2e^{x(2+i3)} - i3e^{x(2+i3)} + 2e^{x(2-i3)} + i3e^{x(2-i3)}\right) =$$

$$\frac{1}{26}\left(2e^{2x}e^{i3x} + 2e^{2x}e^{-i3x} + i3e^{2x}e^{-i3x} - i3e^{2x}e^{i3x}\right) =$$

$$\frac{1}{26}\left(2e^{2x}\left(e^{i3x} + e^{-i3x} + i3e^{-i3x} - i3e^{i3x}\right)\right) =$$

$$\frac{1}{26}\left(2e^{2x}\left(e^{i3x} + e^{-i3x} - i\left(3e^{i3x} - 3e^{-i3x}\right)\right)\right) =$$

$$\frac{e^{2x}}{13}\left(2cos(3x) + 3sin(3x)\right)$$

We check:

$$\frac{d}{dx}\left(\frac{e^{2x}}{13}\left(2cos(3x) + 3sin(3x)\right)\right) =$$

$$2\frac{e^{2x}}{13}\left(2cos(3x) + 3sin(3x)\right) + \frac{e^{2x}}{13}\left(-6sin(3x) + 9cos(3x)\right) =$$

$$2\frac{e^{2x}}{13}\left(2cos(3x) + 3sin(3x)\right) + \frac{e^{2x}}{13}\left(-6sin(3x) + 9cos(3x)\right) =$$

$$\frac{4}{13}e^{2x}cos(3x) + \frac{6}{13}e^{2x}sin(3x) - \frac{6}{13}e^{2x}isn(3x) + \frac{9}{13}e^{2x}cos(3x) =$$

$$\frac{4}{13}e^{2x}cos(3x) + \frac{9}{13}e^{2x}cos(3x) =$$

$$\frac{13}{13}e^{2x}cos(3x) =$$

$$e^{2x}cos(3x)$$