This paper deals with momentum representation and position representation and describe both variants parallel.

A more elaborated text you find in

https://inst.eecs.berkeley.edu/~cs191/fa07/lectures/lecture13_fa07.pdf

Hope I can help you with learning quantum mechanics.

Note: $|\psi\rangle$ is a state (a finite or infinite vector, a Hilbert-space of functions, …)

Quantum mechanics forbids a wave function $\psi(x,p)$ because x and p cannot be known precisely simultaneously. For this reason, we can only work with wave functions $\psi(x)$ and $\tilde{\psi}(p)$.

All of these are eigenfunctions with eigenvalue x or p. They contain the same information about the state $|\psi\rangle$.

Momentum	Position
$|p\rangle \quad \forall p \in \mathbb{R}$ $|p\rangle$ is a **momentum** state in an infinite dimensional vector space, the particle with momentum p. It is a basis of the space of states $|\psi\rangle$. The dimension of $|\psi\rangle$ is infinite. $\tilde{\psi}(p) = \langle p|\psi\rangle$ $\tilde{\psi}(p)$ is an eigenfunction with eigenvalue p.	$|x\rangle \quad \forall x \in \mathbb{R}$ $|x\rangle$ is a **position** state in an infinite dimensional vector space, the particle with position x. It is a basis of the space of states $|\psi\rangle$. The dimension of $|\psi\rangle$ is infinite. $\psi(x) = \langle x|\psi\rangle$ $\psi(x)$ is an eigenfunction with eigenvalue x.
Because $|p\rangle$ is an element of an infinite dimensional vector space, we have: $a|p\rangle \neq |ap\rangle, \qquad a \in \mathbb{R}, a \neq 1$ $-|p\rangle \neq |-p\rangle, \qquad p \neq 0$ $|p_1 + p_2\rangle \neq |p_1\rangle + |p_2\rangle$ Note: $|0\rangle$ is the particle with momentum zero, not a zero vector.	Because $|x\rangle$ is an element of an infinite dimensional vector space, we have: $a|x\rangle \neq |ax\rangle, \qquad a \in \mathbb{R}, a \neq 1$ $-|x\rangle \neq |-x\rangle, \qquad x \neq 0$ $|x_1 + x_2\rangle \neq |x_1\rangle + |x_2\rangle$ Note: $|0\rangle$ is the particle at position zero, not a zero vector.
The inner product: $\langle p_1|p_2\rangle := \delta(p_2 - p_1)$ Different momenta are orthogonal to each other.	The inner product: $\langle x_1|x_2\rangle := \delta(x_2 - x_1)$ Different positions are orthogonal to each other.
The norm of a **momentum** state is infinite: $\langle p|p\rangle := \delta(0) = \infty$	The norm of a **position** state is infinite: $\langle x|x\rangle := \delta(0) = \infty$
Completeness relation: $\mathbf{1} = \int dp\, |p_1\rangle\langle p_1|$ $\mathbf{1}|p_2\rangle = \int dp\, |p_1\rangle\langle p_1|p_2\rangle =$ $\int dp\, |p_1\rangle\, \delta(p_1 - p_2) = |p_2\rangle$	Completeness relation: $\mathbf{1} = \int dx\, |x_1\rangle\langle x_1|$ $\mathbf{1}|x_2\rangle = \int dx\, |x_1\rangle\langle x_1|x_2\rangle =$ $\int dx\, |x_1\rangle\, \delta(x_1 - x_2) = |x_2\rangle$
The momentum operator $\hat{p}$ and position operator $\hat{x}$ are defined by their action on **momentum** states.	The position operator $\hat{x}$ and momentum operator $\hat{p}$ are defined by their action on **position** states.

The eigen-equation for the operator $\hat{p}$: $$\hat{p}\|p\rangle \coloneqq p\|p\rangle$$ In terms of wave functions: $$\hat{p}\psi_p(p) = p\psi_p(p)$$ Result: $$\psi_p(p) = \delta(p - p_0)$$ The eigen-equation for the operator $\hat{x}$: $$\hat{x}\|p\rangle \coloneqq -i\hbar\frac{d}{dp}\|p\rangle = x\|p\rangle$$ In terms of wave-functions: $$\hat{x}\psi_x(p) = -i\hbar\frac{d}{dp}\psi_x(p) = x\psi_x(p)$$ Result of the differential equation: $$\psi_x(p) = \frac{e^{\frac{ipx}{\hbar}}}{\sqrt{2\pi\hbar}}$$ Note: $\frac{1}{\sqrt{2\pi\hbar}}$ is a normalizing factor. Note: $\psi_x(p)$ and $\psi_p(p)$ are functions of p. We labelled them with x and p to make clear that they belong to different eigen-equations (different bases).	The eigen-equation for the operator $\hat{x}$: $$\hat{x}\|x\rangle \coloneqq x\|x\rangle$$ In terms of wave functions: $$\hat{x}\psi_x(x) = x\psi_x(x)$$ Result: $$\psi_x(x) = \delta(x - x_0)$$ The eigen-equation for the operator $\hat{p}$: $$\hat{p}\|x\rangle \coloneqq -i\hbar\frac{d}{dx}\|x\rangle = p\|x\rangle$$ In terms of wave-functions: $$\hat{p}\psi_p(x) = -i\hbar\frac{d}{dx}\psi_p(x) = p\psi_p(x)$$ Result of the differential equation: $$\psi_p(x) = \frac{e^{\frac{ipx}{\hbar}}}{\sqrt{2\pi\hbar}}$$ Note: $\frac{1}{\sqrt{2\pi\hbar}}$ is a normalizing factor. Note: $\psi_p(x)$ and $\psi_x(x)$ are functions of x. We labelled them with p and x to make clear that they belong to different eigen-equations (different bases).
The operator $\hat{x}$ is Hermitian: $$-i\hbar\frac{d}{dp} = i\hbar\frac{d}{dp}$$ *A proof you find at the end of this paper*	The operator $\hat{p}$ is Hermitian: $$-i\hbar\frac{d}{dx} = i\hbar\frac{d}{dx}$$ *A proof you find at the end of this paper*
The operator $\hat{p}$ is Hermitian: $$\hat{p}^\dagger = \hat{p}$$ With this we get: $$\langle p\|\hat{p} = p\langle p\|$$	The operator $\hat{x}$ is Hermitian: $$\hat{x}^\dagger = \hat{x}$$ With this we get: $$\langle x\|\hat{x} = x\langle x\|$$
The wave function associated with a state we get by taking the inner product of a **momentum** state $\|p\rangle$ with the given state$\|\psi\rangle$: $$\tilde{\psi}(p) \coloneqq \langle p\|\psi\rangle \in \mathbb{C}$$ $\langle p\|\psi\rangle$ is a complex number that depends on the value of p, thus a function of p.	The wave function associated with a state we get by taking the inner product of a **position** state $\|x\rangle$ with the given state$\|\psi\rangle$: $$\psi(x) \coloneqq \langle x\|\psi\rangle \in \mathbb{C}$$ $\langle x\|\psi\rangle$ is a complex number that depends on the value of x, thus a function of x.
Any state can be written as a superposition of **momentum** eigenstates: $$\|\psi\rangle = \mathbf{1}\|\psi\rangle = \int dp\ \|p\rangle\langle p\|\psi\rangle\ =$$ $$\int dp\ \|p\rangle\ \tilde{\psi}(p)$$ $\tilde{\psi}(p)$ is the component of $\tilde{\psi}$ along the state $\|p\rangle$.	Any state can be written as a superposition of **position** eigenstates: $$\|\psi\rangle = \mathbf{1}\|\psi\rangle = \int dx\ \|x\rangle\langle x\|\psi\rangle\ =$$ $$\int dx\ \|x\rangle\ \psi(x)$$ $\psi(x)$ is the component of ψ along the state $\|x\rangle$.

The overlap of states (the projection) written in **momentum** space: $$\langle\theta\|\psi\rangle = \langle\theta\|\mathbf{1}\|\psi\rangle = \int dp\, \langle\theta\|p\rangle\langle p\|\psi\rangle = \int dp\, \langle p\|\theta^*\rangle\langle p\|\psi\rangle = \int dp\, \tilde{\theta}^*(p)\, \tilde{\psi}(p)$$	The overlap of states (the projection) written in **position** space: $$\langle\theta\|\psi\rangle = \langle\theta\|\mathbf{1}\|\psi\rangle = \int dx\, \langle\theta\|x\rangle\langle x\|\psi\rangle = \int dx\, \langle x\|\theta^*\rangle\langle x\|\psi\rangle = \int dx\, \theta^*(x)\, \psi(x)$$
Matrix elements of $\hat{p}$: $$\langle\theta\|\hat{p}\|\psi\rangle = \langle\theta\|\hat{p}\mathbf{1}\|\psi\rangle = \int dp\, \langle\theta\|\hat{p}\|p\rangle\langle p\|\psi\rangle = \int dp\, \langle\theta\|p\rangle\, p\, \langle p\|\psi\rangle = \int dp\, \langle p\|\theta^*\rangle\, p\, \langle p\|\psi\rangle = \int dp\, \tilde{\theta}^*(p) p \tilde{\psi}(p)$$	Matrix elements of $\hat{x}$: $$\langle\theta\|\hat{x}\|\psi\rangle = \langle\theta\|\hat{x}\mathbf{1}\|\psi\rangle = \int dx\, \langle\theta\|\hat{x}\|x\rangle\langle x\|\psi\rangle = \int dx\, \langle\theta\|x\rangle\, x\, \langle x\|\psi\rangle = \int dx\, \langle x\|\theta^*\rangle\, x\, \langle x\|\psi\rangle = \int dx\, \theta^*(x) x \psi(x)$$
The overlap $\langle x\|p\rangle$ for a particle with momentum p: $$\langle x\|p\rangle = p(x) = \frac{e^{\frac{ipx}{\hbar}}}{\sqrt{2\pi\hbar}}$$	The overlap $\langle p\|x\rangle$ for a particle with position x: $$\langle p\|x\rangle = x(p) = \frac{e^{-\frac{ipx}{\hbar}}}{\sqrt{2\pi\hbar}}$$
Note: $\langle x\|p\rangle$ and $\langle p\|x\rangle$ differ by a minus sign in the exponent.	
$\tilde{\psi}(p)$ is the Fourier transform of $\psi(x)$: $$\tilde{\psi}(p) = \frac{1}{\sqrt{2\pi\hbar}} \int dx\, e^{-\frac{ipx}{\hbar}} \psi(x)$$ The wave function in the momentum representation $\tilde{\psi}(p)$ is the Fourier transform of the wave function in the position representation $\psi(x)$.	$\psi(x)$ is the Fourier transform of $\tilde{\psi}(p)$: $$\psi(x) = \frac{1}{\sqrt{2\pi\hbar}} \int dp\, e^{\frac{ipx}{\hbar}} \tilde{\psi}(p)$$ The wave function in the position representation $\psi(x)$ is the Fourier transform of the wave function in the momentum representation $\tilde{\psi}(p)$.
We evaluate $\langle p\|\hat{x}\|\psi\rangle$: $$\langle p\|\hat{x}\|\psi\rangle = \int dx\, \langle p\|x\rangle\langle x\|\hat{x}\|\psi\rangle = \int dx\, \langle p\|x\rangle\langle x\|x\|\psi\rangle = \int dx\, \langle p\|x\rangle x\psi(x) = \frac{1}{\sqrt{2\pi\hbar}} \int dx\, e^{-\frac{ipx}{\hbar}} x\psi(x) = i\hbar\left(\frac{-i}{\hbar}\right)\frac{1}{\sqrt{2\pi\hbar}} \int dx\, e^{-\frac{ipx}{\hbar}} x\psi(x) = i\hbar \frac{d}{dp}\tilde{\psi}(p)$$	We evaluate $\langle x\|\hat{p}\|\psi\rangle$: $$\langle x\|\hat{p}\|\psi\rangle = \int dp\, \langle x\|p\rangle\langle p\|\hat{p}\|\psi\rangle = \int dp\, \langle x\|p\rangle\langle p\|p\|\psi\rangle = \int dp\, (p\langle x\|p\rangle)\langle p\|\psi\rangle = \int dp\, \left(\frac{\hbar}{i}\frac{d}{dx}\langle x\|p\rangle\right)\langle p\|\psi\rangle = \frac{\hbar}{i}\frac{d}{dx}\int dp\, \langle x\|p\rangle\langle p\|\psi\rangle = \frac{\hbar}{i}\frac{d}{dx}\langle x\|\psi\rangle = -i\hbar\frac{d}{dx}\, \psi(x)$$

The operator $\hat{p} = -i\hbar\frac{d}{dx}$ is Hermitian. The proof for the operator $\hat{x}$ is analog.

We must show:

$$\int_{-\infty}^{\infty} \psi^* \hat{p} \phi dx = \int_{-\infty}^{\infty} \phi \hat{p}^* \psi^* dx$$

We begin:

$$\int_{-\infty}^{\infty} \psi^* \hat{p} \phi dx = \int_{-\infty}^{\infty} \psi^* \left(-i\hbar \frac{d}{dx}\right) \phi dx = -i\hbar \int_{-\infty}^{\infty} \psi^* d\phi =;$$

We use integration by parts:

$$\int_a^b u dv = [uv]_a^b - \int_a^b v du$$

We proceed:

$$-i\hbar\left([\psi^*\phi]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} \phi d\psi^*\right) =;$$

Note: the wave functions vanish in infinity, so $[\psi^*\phi]_{-\infty}^{\infty}$ becomes zero.

$$= i\hbar \int_{-\infty}^{\infty} \phi d\psi^* = i\hbar \int_{-\infty}^{\infty} \phi \frac{d\psi^*}{dx} dx = \int_{-\infty}^{\infty} \phi \left(i\hbar \frac{d\psi^*}{dx}\right) dx =$$

$$\int_{-\infty}^{\infty} \phi \left(i\hbar \frac{d}{dx}\right) \psi^* dx = \int_{-\infty}^{\infty} \phi \hat{p}^* \psi^* dx$$

Result:

$$\int_{-\infty}^{\infty} \psi^* \hat{p} \phi dx = \int_{-\infty}^{\infty} \phi \hat{p}^* \psi^* dx$$

Note: this works only with functions that are normalizable.

The proof can be found at:

https://www.colby.edu/chemistry/PChem/notes/MomentumHermitian.pdf