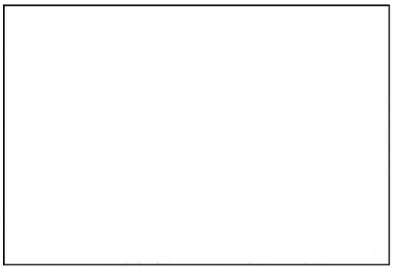
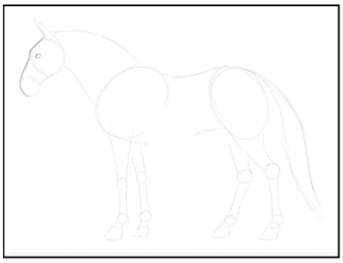
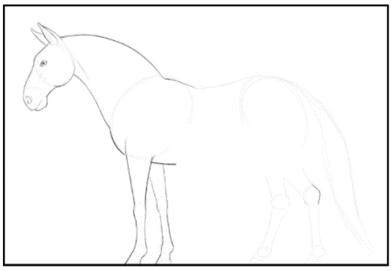
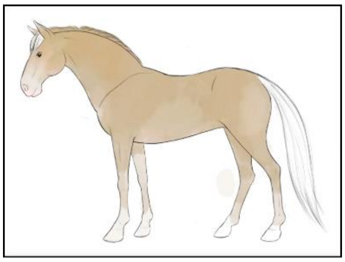
This is an experimental text.

Hope I can help you with learning quantum mechanics.

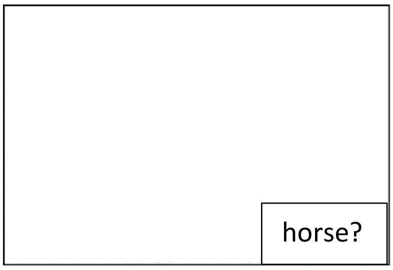
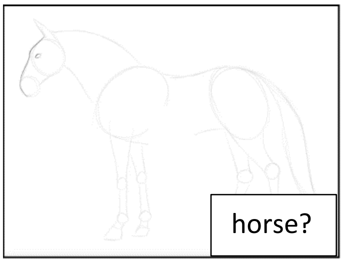
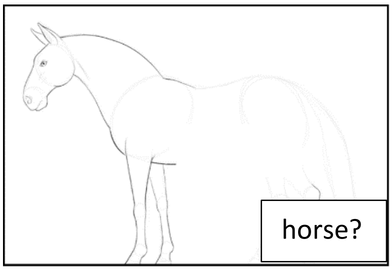
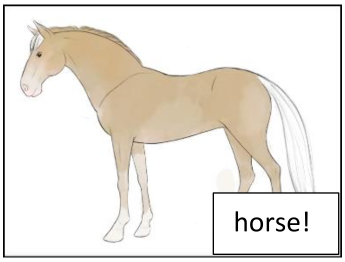
Contents

Painting a Horse

Painting a horse on a canvas starts with an empty canvas:

 The empty canvas	 After some time, there will be a sketch
 The sketch becomes clearer	 The horse undoubtedly becomes a horse

We put a label on the canvas:

 The empty canvas	 After some time, there will be a sketch
 The sketch becomes clearer	 The horse undoubtedly becomes a horse

We use two states.

State one is the no-horse-state.

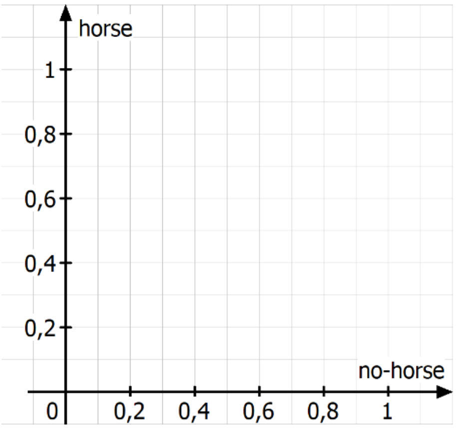
State two is the horse-state.

During the process of painting there will be a mixture of these states.

The picture resembles more or less a horse.

The label requires an "all-or-none" decision: horse or no-horse.

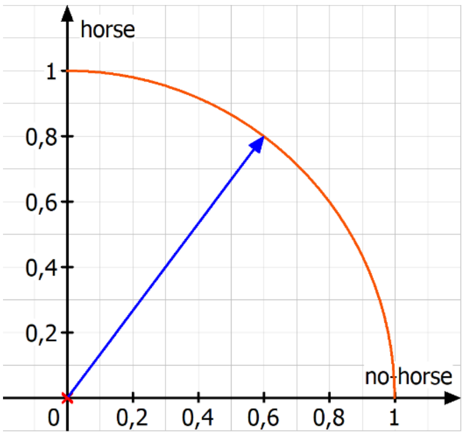
We use vectors to describe this process.



The vector $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ represents the state no-horse.

The vector $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ represents the state horse.

The state of the picture resembles a vector on the unit circle.

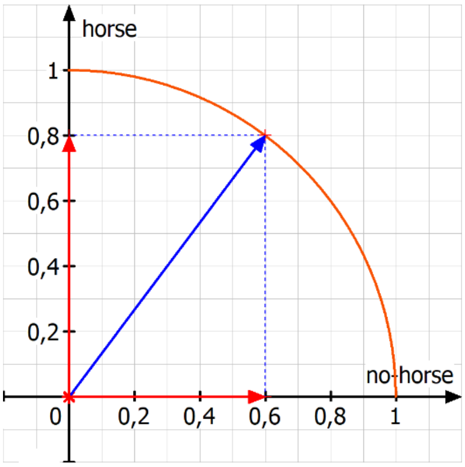


If we ask 100 people whether there is a picture of a horse visible, the empty canvas will 100% produce the answer no.

If we ask 100 people whether there is a picture of a horse visible, the completed picture of the horse will 100% produce the answer yes.

In between we will get percentages.

The unit vector swinging from the no-horse axis to the horse-axis can be used.



The state above shows an example where 36% will state "no-horse" and 64% will state "horse".

In 2-D vector geometry we write the state of the painting $\binom{x}{y}$ as:

$$\binom{x}{y} = 0.6\binom{1}{0} + 0.8\binom{0}{1} = \binom{0.6}{0.8}$$

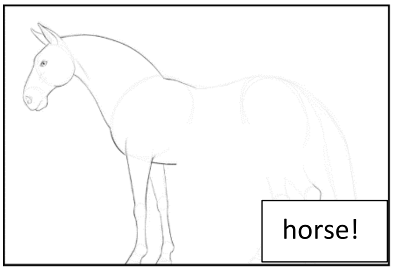
The state vector describes whether there is a horse visible or not.

In general, we write:

$$\binom{x}{y} = a\binom{1}{0} + b\binom{0}{1}, a^2 + b^2 = 1$$

The label on the canvas alters the situation.

If we got a decision by asking some spectators "horse" or "no-horse", we fix it on the label.



From now on all spectators of the picture follow the label and decide "horse".

The state vector get a fixed value:

$$\binom{x}{y} = 0\binom{1}{0} + 1\binom{0}{1} = \binom{0}{1}$$

This holds as long the inscription on the label is visible.

Unfortunately, we used bad chalk for the label that washes away if the wind blows.

Painting two pictures

If we have two artists painting on two canvas, we can treat each picture separately. This is the case of classical physics.

We use the direct sum of two vectors.

Painting one gives:

$$a\begin{pmatrix}1\\0\end{pmatrix}+b\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}a\\b\end{pmatrix}, a^2+b^2=1$$

Painting two gives:

$$c\begin{pmatrix}1\\0\end{pmatrix}+d\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}c\\d\end{pmatrix}, c^2+d^2=1$$

The direct sum:

$$\begin{pmatrix}a\\b\end{pmatrix}\oplus\begin{pmatrix}c\\d\end{pmatrix}=\begin{pmatrix}a\\b\\c\\d\end{pmatrix}, a^2+b^2=1=c^2+d^2$$

$\begin{pmatrix}a\\b\\c\\d\end{pmatrix}$ is the combined state. It consists of two separate states. We can build any combined state by building the direct sum of two vectors $\begin{pmatrix}a\\b\end{pmatrix}\oplus\begin{pmatrix}c\\d\end{pmatrix}$.

Painting on one canvas

We have two artists painting on one canvas, using different colors, red and green. Maybe the first artist paints a horse using red color. The second artist paints a lion using green color. Unfortunately, we are red-green color blind and see them as gray. If we watch the canvas, we see a mixture of gray lines we cannot separate.

We describe the situation with the tensor product.

The "red eye" gives:

$$\begin{pmatrix}w\\x\end{pmatrix}=a\begin{pmatrix}1\\0\end{pmatrix}+b\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}a\\b\end{pmatrix}, a^2+b^2=1$$

The "green eye" gives:

$$\begin{pmatrix}y\\z\end{pmatrix}=c\begin{pmatrix}1\\0\end{pmatrix}+d\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}c\\d\end{pmatrix}, c^2+d^2=1$$

The combined state:

$$\vec{v}:=\begin{pmatrix}w\\x\\y\\z\end{pmatrix}=\begin{pmatrix}a\\b\end{pmatrix}\otimes\begin{pmatrix}c\\d\end{pmatrix}=\begin{pmatrix}ac\\ad\\bc\\bd\end{pmatrix}, a^2+b^2=1=c^2+d^2$$

The tensor product brings new behavior.

1. There are vectors $\vec{v}$ in the combined state that can be assembled by the tensor product of vectors $\binom{a}{b} \otimes \binom{c}{d}$.
2. But there are vectors $\vec{v}$ in the combined state that cannot be assembled by the tensor product of vectors $\binom{a}{b} \otimes \binom{c}{d}$. The combined space contains more possibilities than any combination of the two basic states.

We check this.

For 1. we use:

$$\vec{v} \coloneqq \frac{1}{2}\begin{pmatrix}1\\1\\1\\1\end{pmatrix}$$

We choose the vectors $\vec{g}$ and $\vec{h}$:

$$\vec{g} \coloneqq \sqrt{\frac{1}{2}}\binom{1}{0} + \sqrt{\frac{1}{2}}\binom{0}{1} = \sqrt{\frac{1}{2}}\binom{1}{1}$$

$$\vec{h} \coloneqq \sqrt{\frac{1}{2}}\binom{1}{0} + \sqrt{\frac{1}{2}}\binom{0}{1} = \sqrt{\frac{1}{2}}\binom{1}{1}$$

Obviously $\vec{g}$ and $\vec{h}$ are valid state vectors. We build the tensor product $\vec{g}\otimes\vec{h}$:

$$\vec{g}\otimes\vec{h} = \sqrt{\frac{1}{2}}\binom{1}{1} \otimes \sqrt{\frac{1}{2}}\binom{1}{1} = \frac{1}{2}\begin{pmatrix}1\\1\\1\\1\end{pmatrix}$$

Result: The vector $\vec{v} \coloneqq \frac{1}{2}\begin{pmatrix}1\\-1\\-1\\1\end{pmatrix}$ of the combined space can be assembled by the tensor product of the two states $\vec{g}\otimes\vec{h}$.

For 2. we use:

$$\vec{u} \coloneqq \begin{pmatrix}1\\0\\0\\1\end{pmatrix}$$

We try to assemble vector $\vec{u}$ by a combination of vectors of the basis states:

$$\vec{u} = \left(a\binom{1}{0} + b\binom{0}{1}\right) \otimes \left(c\binom{1}{0} + d\binom{0}{1}\right)$$

We build the tensor product:

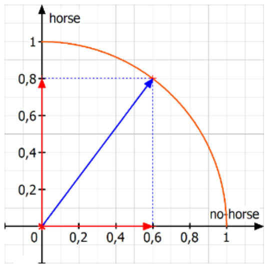
$$\vec{u} = \left(\binom{a}{b}\right) \otimes \left(\binom{c}{d}\right) = \begin{pmatrix}ac\\ad\\bc\\bd\end{pmatrix}$$

We check the coefficients:

$$ac = 1; ad = 0; bc = 0; bd = 1$$

From $ad = 0$ we get that either a or d must be zero, so either ac or bd cannot be 1.

Why?



The valid states represent a curve, an object with one dimension. The combination of two curves reach every point in a plane. On the other hand, the tensor product brings up an object with four dimensions. There must be points in this object that cannot be reached by a plane.