Sometimes we need to do some arithmetic with bras and kets of different dimension.

On the conceptual level we have the following rules:

$$\langle 0|00\rangle = \langle 0|0\rangle|0\rangle = 1\cdot|0\rangle = |0\rangle$$

$$\langle 0|01\rangle = \langle 0|0\rangle|1\rangle = 1\cdot|1\rangle = |1\rangle$$

$$\langle 0|10\rangle = \langle 0|1\rangle|0\rangle = 0\cdot|0\rangle = 0$$

$$\langle 0|11\rangle = \langle 0|1\rangle|1\rangle = 0\cdot|1\rangle = 0$$

$$\langle 1|00\rangle = \langle 1|0\rangle|0\rangle = 0\cdot|0\rangle = 0$$

$$\langle 1|01\rangle = \langle 1|0\rangle|1\rangle = 0\cdot|1\rangle = 0$$

$$\langle 1|10\rangle = \langle 1|1\rangle|0\rangle = 1\cdot|0\rangle = |0\rangle$$

$$\langle 1|11\rangle = \langle 1|1\rangle|1\rangle = 1\cdot|1\rangle = |1\rangle$$

Note: similar we have:

$$\langle 00|0\rangle = \langle 0|\langle 0|0\rangle = \langle 0|\cdot 1 = \langle 0|$$

$$\dots\ etc.$$

On the basic level we resolve the Kronecker product. We remember:

$\langle 0| = (1\ 0)$ $\langle 1| = (0\ 1)$	$|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ $|1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$
$\langle 00| = (1\ 0)\otimes(1\ 0) = (1\ 0\ 0\ 0)$ $\langle 01| = (1\ 0)\otimes(0\ 1) = (0\ 1\ 0\ 0)$ $\langle 10| = (0\ 1)\otimes(1\ 0) = (0\ 0\ 1\ 0)$ $\langle 11| = (0\ 1)\otimes(0\ 1) = (0\ 0\ 0\ 1)$	$|00\rangle = \begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$ $|01\rangle = \begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}$ $|10\rangle = \begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\0\\1\\0\end{pmatrix}$ $|11\rangle = \begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$

We calculate:

$$\langle 0|00\rangle = (1\ 0)\cdot\begin{pmatrix}1\\0\\0\\0\end{pmatrix} =?$$

This scalar product can't be calculated because the dimensions do not fit. We note that $|00\rangle$ is not a fully 4-dimensional vector but instead the Kronecker product of two 2-dimensional vectors so we can replace it.

$$\langle 0|00\rangle = (1\ 0)\cdot\begin{pmatrix}1\\0\\0\\0\end{pmatrix} = (1\ 0)\cdot\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix} = 1\otimes\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$$

$$\langle 0|01\rangle = (1\ 0)\cdot\begin{pmatrix}0\\1\\0\\0\end{pmatrix} = (1\ 0)\cdot\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}0\\1\end{pmatrix} = 1\otimes\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}$$

$$\langle 0|10\rangle = (1\ 0)\cdot\begin{pmatrix}0\\0\\1\\0\end{pmatrix} = (1\ 0)\cdot\begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix} = 0\otimes\begin{pmatrix}1\\0\end{pmatrix} = 0$$

$$\langle 0|11\rangle = (1\ 0)\cdot\begin{pmatrix}0\\0\\0\\1\end{pmatrix} = (1\ 0)\cdot\begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}0\\1\end{pmatrix} = 0\otimes\begin{pmatrix}0\\1\end{pmatrix} = 0$$

$$\langle 1|00\rangle = (0\ 1)\cdot\begin{pmatrix}1\\0\\0\\0\end{pmatrix} = (0\ 1)\cdot\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix} = 0\otimes\begin{pmatrix}1\\0\end{pmatrix} = 0$$

$$\langle 1|01\rangle = (0\ 1)\cdot\begin{pmatrix}0\\1\\0\\0\end{pmatrix} = (0\ 1)\cdot\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}0\\1\end{pmatrix} = 0\otimes\begin{pmatrix}0\\1\end{pmatrix} = 0$$

$$\langle 1|10\rangle = (0\ 1)\cdot\begin{pmatrix}0\\0\\1\\0\end{pmatrix} = (0\ 1)\cdot\begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix} = 1\otimes\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$$

$$\langle 1|11\rangle = (0\ 1)\cdot\begin{pmatrix}0\\0\\0\\1\end{pmatrix} = (0\ 1)\cdot\begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}0\\1\end{pmatrix} = 1\otimes\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}$$

Note: similar we have:

$$\langle 00|0\rangle = (1\ 0\ 0\ 0)\cdot\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}\cdot(1\ 0) = \begin{pmatrix}1\\0\end{pmatrix}\otimes 1 = \begin{pmatrix}1\\0\end{pmatrix}$$

$$\ldots etc.$$

With this scheme we can resolve any kind of $\langle m|n\rangle$ partial scalar products.

Note: $a\otimes\begin{pmatrix}0\\1\end{pmatrix} = a\cdot\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\a\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}\cdot a = \begin{pmatrix}0\\1\end{pmatrix}\otimes a$