We have a combination of three qubits. Qubit one, $|\psi\rangle$ and an entangled pair of qubits $|\phi^+\rangle$, one of the Bell states:

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

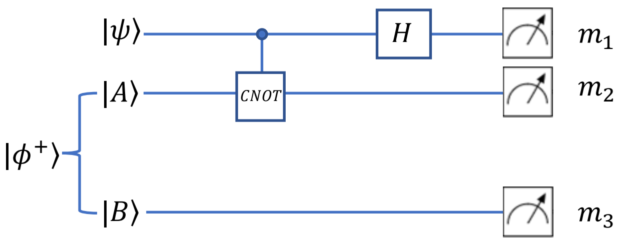
We bring qubit one, $|\psi\rangle$ into the definite state $0.5|0\rangle + 0.8660245|1\rangle$ by help of the expression

ry(2 * pi / 3) q[0];

We call the parameters α and β.

Note: You find this rotation (initializing) at:
https://quantumcomputing.stackexchange.com/questions/16501/how-to-initialize-a-qubit-with-a-custom-state-in-qiskit-composer

We perform a CNOT from $|\psi\rangle$ to $|A\rangle$ and a Hadamard on the qubit $|\psi\rangle$ and get a superposition of all three qubits:

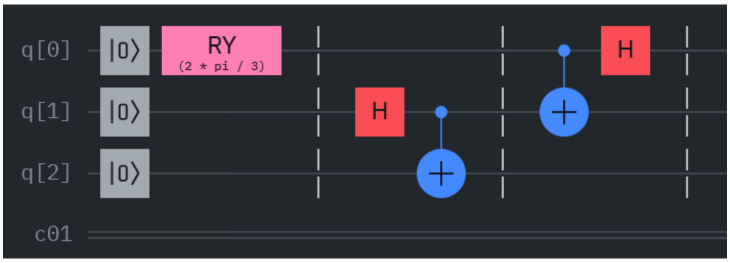


Note:

Denote the Hadamard gate

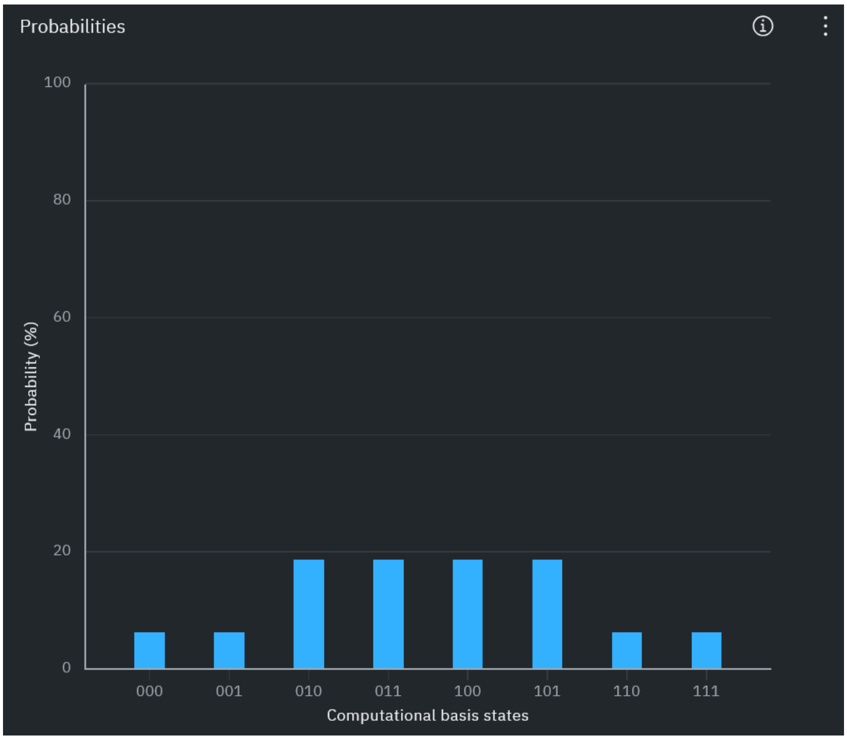
Denote a controlled Not gate

In the IBM composer this looks like:



Note: The first Hadamard and CNOT produce the Bell-state $|\phi^+\rangle$ from qubits $|A\rangle$ and $|B\rangle$.

We get the probabilities for the computational basis states:



Let us take a closer look at the scenario.

The initial state of the qubits $|\psi\rangle$ and $|\phi^+\rangle$ we name $|\pi_0\rangle$.

In detail:

$|\pi_0\rangle = |\phi^+\rangle|\psi\rangle$ resp. $|\phi^+\psi\rangle$

The qubit $|\psi\rangle$ is in the state $\alpha|0\rangle + \beta|1\rangle$.

We get:

$$|\pi_0\rangle = |\phi^+\rangle(\alpha|0\rangle + \beta|1\rangle)$$

We expand $|\phi^+\rangle$:

$$|\pi_0\rangle = \left(\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)\right)(\alpha|0\rangle + \beta|1\rangle) =$$

$$\frac{\alpha|000\rangle + \alpha|110\rangle + \beta|001\rangle + \beta|111\rangle}{\sqrt{2}}$$

We apply the $CNOT$ on $|\pi_0\rangle$ and get $|\pi_1\rangle$.

Note that the $CNOT$ is controlled by the last qubit and acts on the second qubit (as we use IBM notation):

$$|\pi_1\rangle = \frac{\alpha|000\rangle + \alpha|110\rangle + \beta|011\rangle + \beta|101\rangle}{\sqrt{2}}$$

We rewrite:

$$|\pi_1\rangle = \frac{\alpha|000\rangle + \alpha|110\rangle + \beta|011\rangle + \beta|101\rangle}{\sqrt{2}} = \frac{\alpha|00\rangle|0\rangle + \alpha|11\rangle|0\rangle + \beta|01\rangle|1\rangle + \beta|10\rangle|1\rangle}{\sqrt{2}} =$$

$$\frac{(\alpha|00\rangle + \alpha|11\rangle)|0\rangle + (\beta|01\rangle + \beta|10\rangle|1\rangle}{\sqrt{2}}$$

We apply the Hadamard on $|\psi\rangle$:

$$H|\pi_1\rangle = \frac{(\alpha|00\rangle + \alpha|11\rangle)\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) + (\beta|01\rangle + \beta|10\rangle\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)}{\sqrt{2}} =$$

$$\frac{(\alpha|00\rangle + \alpha|11\rangle)(|0\rangle + |1\rangle) + (\beta|01\rangle + \beta|10\rangle(|0\rangle - |1\rangle)}{2} =$$

$$\frac{\alpha|000\rangle + \alpha|001\rangle + \alpha|110\rangle + \alpha|111\rangle + \beta|010\rangle - \beta|011\rangle + \beta|100\rangle - \beta|101\rangle}{2} =$$

$$\frac{\alpha|000\rangle + \alpha|001\rangle + \beta|010\rangle - \beta|011\rangle + \beta|100\rangle - \beta|101\rangle + \alpha|110\rangle + \alpha|111\rangle}{2} = \pi_2$$

This is the state after the $CNOT$ and the Hadamard. We express this as a state vector:

$$\frac{1}{2} \cdot \begin{pmatrix} \alpha \\ \alpha \\ \beta \\ -\beta \\ \beta \\ -\beta \\ \alpha \\ \alpha \end{pmatrix}$$

Now we are measuring.

We measure the first qubit m_1. Note that this acts on the last of our three qubits (we use IBM notation).

Conceptional view:

$$\left(\frac{1}{2}\right) \cdot (\alpha|000\rangle + \alpha|001\rangle + \alpha|110\rangle + \alpha|111\rangle + \beta|010\rangle - \beta|011\rangle + \beta|100\rangle - \beta|101\rangle) =$$

$$\left(\frac{1}{2}\right) \cdot (\alpha|000\rangle + \alpha|110\rangle + \beta|010\rangle + \beta|100\rangle + \alpha|001\rangle + \alpha|111\rangle - \beta|011\rangle - \beta|101\rangle) =$$

$$\left(\frac{1}{2}\right) \cdot (\alpha|000\rangle + \alpha|110\rangle + \beta|010\rangle + \beta|100\rangle) + \left(\frac{1}{2}\right) \cdot (\alpha|001\rangle + \alpha|111\rangle - \beta|011\rangle - \beta|101\rangle) =$$

$$\left(\frac{1}{2}\right) \cdot (\alpha|00\rangle + \alpha|11\rangle + \beta|01\rangle + \beta|10\rangle)|0\rangle + \left(\frac{1}{2}\right) \cdot (\alpha|00\rangle + \alpha|11\rangle - \beta|01\rangle - \beta|10\rangle)|1\rangle =$$

$$\left(\frac{1}{2}\right) \cdot (\alpha|00\rangle + \beta|01\rangle + \beta|10\rangle + \alpha|11\rangle)|0\rangle + \left(\frac{1}{2}\right) \cdot (\alpha|00\rangle - \beta|01\rangle - \beta|10\rangle + \alpha|11\rangle)|1\rangle$$

State vector view:

$$\frac{1}{2}\left(\alpha\begin{pmatrix}1\\0\\0\\0\end{pmatrix}+\beta\begin{pmatrix}0\\1\\0\\0\end{pmatrix}+\beta\begin{pmatrix}0\\0\\1\\0\end{pmatrix}+\alpha\begin{pmatrix}0\\0\\0\\1\end{pmatrix}\right)\otimes\begin{pmatrix}1\\0\end{pmatrix}+\frac{1}{2}\left(\alpha\begin{pmatrix}1\\0\\0\\0\end{pmatrix}-\beta\begin{pmatrix}0\\1\\0\\0\end{pmatrix}-\beta\begin{pmatrix}0\\0\\1\\0\end{pmatrix}+\alpha\begin{pmatrix}0\\0\\0\\1\end{pmatrix}\right)\otimes\begin{pmatrix}0\\1\end{pmatrix}=$$

$$\frac{1}{2}\cdot\left(\left(\begin{pmatrix}\alpha\\\beta\\\beta\\\alpha\end{pmatrix}\right)\otimes\begin{pmatrix}1\\0\end{pmatrix}+\left(\begin{pmatrix}\alpha\\-\beta\\-\beta\\\alpha\end{pmatrix}\right)\otimes\begin{pmatrix}0\\1\end{pmatrix}\right)$$

What has happened: The 8D state vector collapses into two possible 4D state vectors:

$$\begin{pmatrix}\alpha\\\beta\\\beta\\\alpha\end{pmatrix}\text{ or }\begin{pmatrix}\alpha\\-\beta\\-\beta\\\alpha\end{pmatrix}$$

Measuring in the standard basis acts like a comb that combs out every second entry:

$$\begin{pmatrix}\alpha\\-\beta\\-\beta\\\alpha\end{pmatrix}\longleftarrow\begin{pmatrix}\alpha\\\alpha\\\beta\\-\beta\\\beta\\-\beta\\\alpha\\\alpha\end{pmatrix}\longrightarrow\begin{pmatrix}\alpha\\\beta\\\beta\\\alpha\end{pmatrix}$$

After the measurement we get either the state $\begin{pmatrix}\alpha\\\beta\\\beta\\\alpha\end{pmatrix}$ or the state $\begin{pmatrix}\alpha\\-\beta\\-\beta\\\alpha\end{pmatrix}$.

We measure the second qubit. Note that again this is the right qubit in IBM notation.

In the conceptual view:

$\left(\frac{1}{2}\right)\cdot(\alpha\|00\rangle+\beta\|01\rangle+\beta\|10\rangle+\alpha\|11\rangle)=$	$\left(\frac{1}{2}\right)\cdot(\alpha\|00\rangle-\beta\|01\rangle-\beta\|10\rangle+\alpha\|11\rangle)=$
$\left(\frac{1}{2}\right)\cdot(\alpha\|00\rangle+\beta\|10\rangle+\beta\|01\rangle+\alpha\|11\rangle)=$	$\left(\frac{1}{2}\right)\cdot(\alpha\|00\rangle-\beta\|10\rangle-\beta\|01\rangle+\alpha\|11\rangle)=$
$\left(\frac{1}{2}\right)\cdot\big((\alpha\|0\rangle+\beta\|1\rangle)\|0\rangle+(\beta\|0\rangle+\alpha\|1\rangle)\|1\rangle\big)$	$\left(\frac{1}{2}\right)\cdot\big((\alpha\|0\rangle-\beta\|1\rangle)\|0\rangle+(-\beta\|0\rangle+\alpha\|1\rangle)\|1\rangle\big)$

In case we measure $|0\rangle$ we get either $\alpha|0\rangle+\beta|1\rangle$ or $\alpha|0\rangle-\beta|1\rangle$.

In case we measure $|1\rangle$ we get either $\beta|0\rangle+\alpha|1\rangle$ or $-\beta|0\rangle+\alpha|1\rangle$.

The probabilities are the same.

In state vector view:

$\left(\frac{1}{2}\right)\left((\alpha\|0\rangle+\beta\|1\rangle)\|0\rangle+(\beta\|0\rangle+\alpha\|1\rangle)\|1\rangle\right)\rightarrow$ $\frac{1}{2}\left(\alpha\begin{pmatrix}1\\0\end{pmatrix}+\beta\begin{pmatrix}0\\1\end{pmatrix}\right)\otimes\begin{pmatrix}1\\0\end{pmatrix}$ $+\frac{1}{2}\left(\beta\begin{pmatrix}1\\0\end{pmatrix}+\alpha\begin{pmatrix}0\\1\end{pmatrix}\right)\otimes\begin{pmatrix}0\\1\end{pmatrix}$	$\left(\frac{1}{2}\right)\left((\alpha\|0\rangle-\beta\|1\rangle)\|0\rangle+(-\beta\|0\rangle+\alpha\|1\rangle)\|1\rangle\right)\rightarrow$ $\frac{1}{2}\left(\alpha\begin{pmatrix}1\\0\end{pmatrix}-\beta\begin{pmatrix}0\\1\end{pmatrix}\right)\otimes\begin{pmatrix}1\\0\end{pmatrix}$ $+\frac{1}{2}\left(-\beta\begin{pmatrix}1\\0\end{pmatrix}+\alpha\begin{pmatrix}0\\1\end{pmatrix}\right)\otimes\begin{pmatrix}0\\1\end{pmatrix}$

The 4D state vector collapses with equal probability to:

$\frac{1}{2}\begin{pmatrix}\alpha\\\beta\end{pmatrix}$ or $\frac{1}{2}\begin{pmatrix}\beta\\\alpha\end{pmatrix}$	$\frac{1}{2}\begin{pmatrix}\alpha\\-\beta\end{pmatrix}$ or $\frac{1}{2}\begin{pmatrix}-\beta\\\alpha\end{pmatrix}$

In detail:

If the first measurement gives $\begin{pmatrix}1\\0\end{pmatrix}$, the system collapses to $\begin{pmatrix}\alpha\\\beta\\\beta\\\alpha\end{pmatrix}$	
The second measurement gives $\begin{pmatrix}1\\0\end{pmatrix}$, the system collapses to $\begin{pmatrix}\alpha\\\beta\end{pmatrix}$.	The second measurement gives $\begin{pmatrix}0\\1\end{pmatrix}$, the system collapses to $\begin{pmatrix}\beta\\\alpha\end{pmatrix}$.

If the first measurement gives $\begin{pmatrix}0\\1\end{pmatrix}$, the system collapses to $\begin{pmatrix}\alpha\\-\beta\\-\beta\\\alpha\end{pmatrix}$	
The second measurement gives $\begin{pmatrix}1\\0\end{pmatrix}$, the system collapses to $\begin{pmatrix}\alpha\\-\beta\end{pmatrix}$.	The second measurement gives $\begin{pmatrix}0\\1\end{pmatrix}$, the system collapses to $\begin{pmatrix}-\beta\\\alpha\end{pmatrix}$.

From this we learn:

- If the result of the measurements is $\begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}$, then the remaining state vector is $\begin{pmatrix}\alpha\\\beta\end{pmatrix}$.
- If the result of the measurements is $\begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}$, then the remaining state vector is $\begin{pmatrix}\beta\\\alpha\end{pmatrix}$.
- If the result of the measurements is $\begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}$, then the remaining state vector is $\begin{pmatrix}\alpha\\-\beta\end{pmatrix}$
- If the result of the measurements is $\begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}$, then the remaining state vector is $\begin{pmatrix}-\beta\\\alpha\end{pmatrix}$

In the first case we need not do anything, the remaining qubit is the one Alice had.

In the second case we need to flip the qubit.

In the third case we need a phase shift on the second entry.

In the fourth case we need both, a flip and a phase shift on the second entry.

We see that Bob needs information about the result of the measurement to be able to restore Alice's qubit correctly.

Note: the collapse of the 4D vectors happens in the same "comb out" manner as the collapse of the 8D vector. Due to only two values α and β this is somewhat disguised:

$\begin{pmatrix} \alpha \\ \beta \\ \beta \\ \alpha \end{pmatrix}$	$\begin{pmatrix} \alpha \\ \beta \\ \beta \\ \alpha \end{pmatrix}$