Most classical gates are one-way. After processing the input is not fully recoverable.

Truth-table for “and”:

A	B	A and B
0	0	0
0	1	0
1	0	0
1	1	1

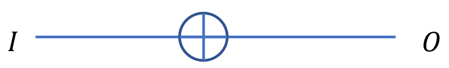
Only in case the output equals 1 we know that both inputs are 1 too. In case the output is 0 we cannot determine the state of inputs A or B.

The NOT-gate is reversible:

A	$\overline{A}$
0	1
1	0

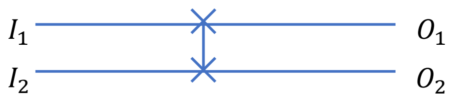
Quantum gates need to be reversible. Each output must correspond to a specific input. The following are reversible quantum gates.

NOT



I	O
0	1
1	0

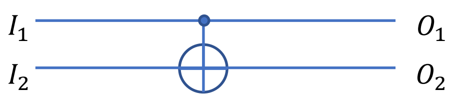
SWAP



I_1	I_2	O_1	O_2
0	0	0	0
0	1	1	0
1	0	0	1
1	1	1	1

Note: Swap only visible if $I_1 \neq I_2$.

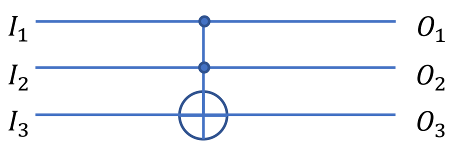
CNOT



I_1	I_2	O_1	O_2
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

The CNOT negates I_2 if $I_1 = 1$.

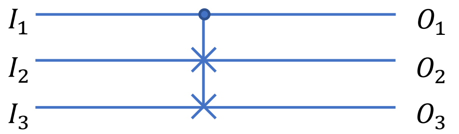
TOFFOLI



I_1	I_2	I_3	O_1	O_2	O_3
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	0	1
1	1	0	1	1	1
1	1	1	1	1	0

The Toffoli-gate negates I_3 if $I_1 = I_2 = 1$.

FREDKIN



If $I_1 = 1$ the Fredkin-gate swaps I_2 and I_3 . The effect is visible only if $I_2 \neq I_3$.

I_1	I_2	I_3	O_1	O_2	O_3
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	1	0
1	1	0	1	0	1
1	1	1	1	1	1

We can put this in other words.

The FREDKIN-gate swaps $I_2\ and\ I_3$ if $I_1 = 1$, a controlled swap.

The TOFFOLI-gate negates I_3 if I_1 and I_2 both are 1, a double controlled not.

The CNOT-gate negates I_2 if $I_1 = 1$, a controlled not.

The SWAP and the NOT do what they are expected to do.

Note: The gates are reversible because we have unique combinations on input-side and output-side.

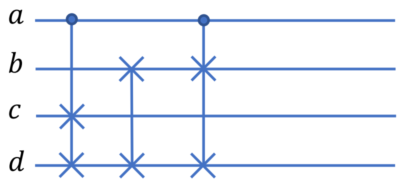
Example

For easier reading we name the lines a, b, c, d and omit the distinctions between input and output.

We want a combination that

- swaps lines b and d if line $a = 0$ and
- swaps lines c and d if line $a = 1$ and
- leaves line a untouched

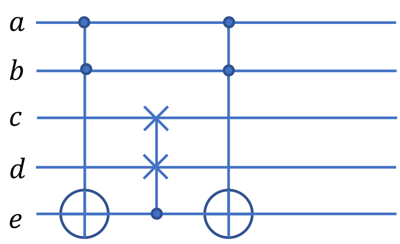
input				output			
a	b	c	d	a	b	c	d
0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0
0	1	0	0	0	0	0	1
1	1	0	0	1	1	0	0
0	0	1	0	0	0	1	0
1	0	1	0	1	0	0	1
0	1	1	0	0	0	1	1
1	1	1	0	1	1	0	1
0	0	0	1	0	1	0	0
1	0	0	1	1	0	1	0
0	1	0	1	0	1	0	1
1	1	0	1	1	1	1	0
0	0	1	1	0	1	1	0
1	0	1	1	1	0	1	1
0	1	1	1	0	1	1	1
1	1	1	1	1	1	1	1



Example

We want a combination that

- swaps lines c and d if line $a = b = 1$



We need an auxiliary line e to perform this.

input				output			
a	b	c	d	a	b	c	d
0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0
0	1	0	0	0	1	0	0
1	1	0	0	1	1	0	0
0	0	1	0	0	0	1	0
1	0	1	0	1	0	1	0
0	1	1	0	0	1	1	0
1	1	1	0	1	1	0	1
0	0	0	1	0	0	0	1
1	0	0	1	1	0	0	1
0	1	0	1	0	1	0	1
1	1	0	1	1	1	1	0
0	0	1	1	0	0	1	1
1	0	1	1	1	0	1	1
0	1	1	1	0	1	1	1
1	1	1	1	1	1	1	1