How to realize a spin flip.

This is an excerpt of a lecture hold by Wolfgang Ketterle dating from the year 2014.

You may find the whole course at:

https://mitxonline.mit.edu/courses/course-v1:MITxT+8.421x/

Hope I can help you with learning quantum mechanics.

Magnetic Moment in a Static Field

The interaction energy W of a magnetic moment $\vec{\mu}$ with a magnetic field $\vec{B}$ is given by:

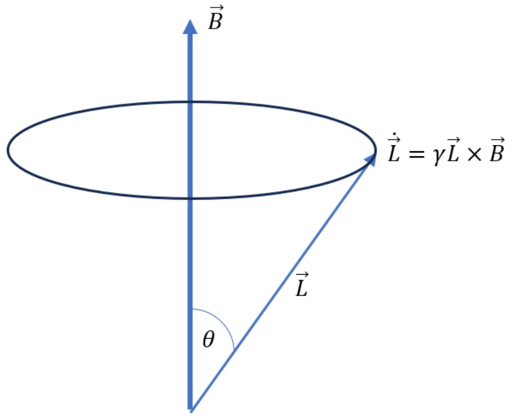
$$W = -\vec{\mu} \cdot \vec{B} = -|\vec{\mu}| \cdot |\vec{B}| \cdot cos(\theta)$$

In a uniform field the force $\vec{F} = -\nabla W$ vanishes, but the torque $\vec{T} = \vec{\mu} \times \vec{B}$ does not.

For an angular momentum $\vec{L}$ we get the equation of motion:

$$\dot{\vec{L}} = \vec{T} = \vec{\mu} \times \vec{B} = \gamma \vec{L} \times \vec{B}$$

Note: the gyromagnetic ratio $\gamma = \frac{Q}{2m}$ is the proportionality constant between angular momentum and magnetic moment.



The magnetic moment $\vec{\mu}$ is precessing around the magnetic field $\vec{B}$ with angular frequency:

$$\Omega_L = -\gamma |\vec{B}| = \frac{e}{2m} |\vec{B}|$$

Note: Ω_L is the Larmor frequency.

For electrons we have $\gamma_e = 2\pi \cdot 2.8 \frac{MHz}{Gauss}$, for protons $\gamma_p = 2\pi \cdot 4.2 \frac{KHz}{Gauss}$.

Rotating Magnetic Field on Resonance

Consider a magnetic moment $\vec{\mu}$ precessing around a field $\vec{B} = |\vec{B}_0| \hat{e}_z$.

We use:

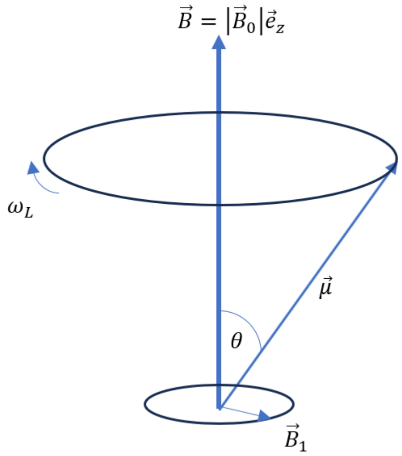
$$|\vec{\mu}| = \text{const.}, \theta = \text{const.}, \phi(t) = -\omega_L t \text{ with } \omega_L = -\gamma |\vec{B}_0|$$

as shown in the figure below.

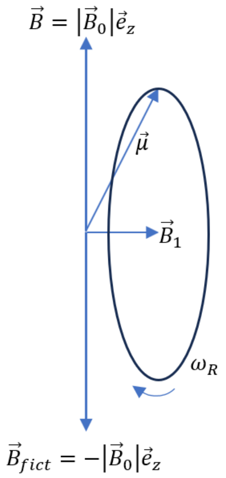
Note: $\phi(t)$ is the rotation of the precession around the $\vec{e}_z$-axis.

Assume that we apply an additional rotating magnetic field $\vec{B}_1$ in the xy -plane, rotating at ω_0.

We look at this problem in the rotating coordinate system.



Inertial system



rotating frame

There $\vec{B}_1$ becomes stationary, e. g. along $\vec{e}_x$ and it appears an additional fictitious field:

$$\left|\vec{B}_{fict}\right| = \frac{\omega_0}{\gamma} = -\left|\vec{B}_0\right|$$

The fictitious field is aligned with the field $\vec{B}_0$, antiparallel. The fictitious field cancels the field $\vec{B}_0$.

In the rotating frame we are left with the, then, static field $\vec{B}_1$.

The magnetic moment $\vec{\mu}$ precesses around $\vec{B}_1$ with the Rabi frequency

$$\omega_R = \gamma\left|\vec{B}_1\right|$$

This precession from time to time leads to a spin flip.

At time $t = 0$ we have $\vec{\mu} = |\vec{\mu}|\vec{e}_z$

This magnetic moment initially along the $\vec{e}_z$ axis will flip to the $-\vec{e}_z$ axis after a time t, the so-called π-pulse:

$$t = \frac{\pi}{\omega_R}$$

This method works but is not easy to handle because the Rabi frequencies are in the range of MHz for electrons.

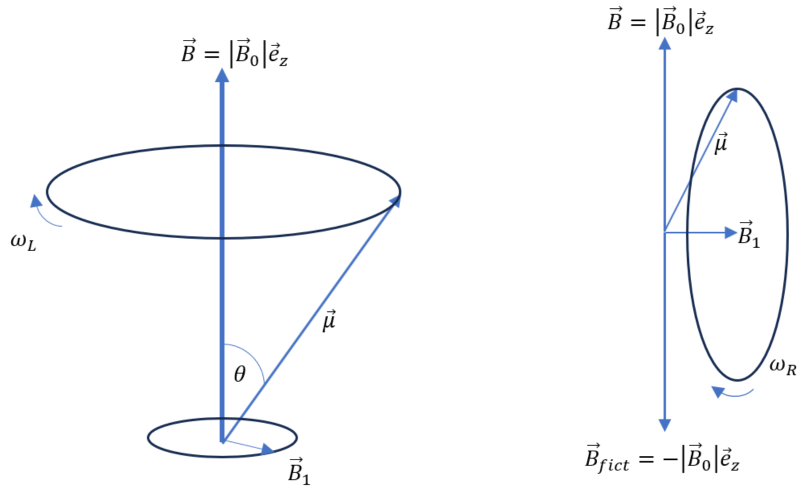
We need a method to perform a controlled spin flip.

"Rapid" Adiabatic Passage

Rapid adiabatic passage is a technique for inverting a spin by slowly sweeping the detuning of a drive field through resonance.

Slowly with respect to the Larmor frequency $\gamma \cdot \left|\vec{B}_{eff}\right|$ around the effective static field in the rotating frame.

We remember the picture:



Inertial system rotating frame

This picture is valid if the magnetic field $\vec{B}_1$ rotates with $\omega_0 = -\gamma|\vec{B}_0|$.

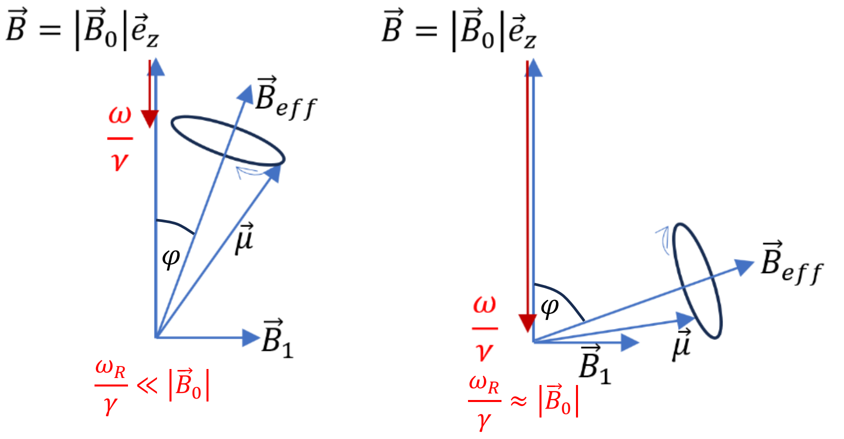
We slightly detune ω_0:

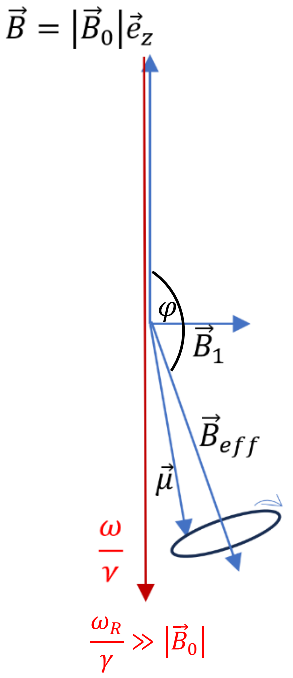
$$\delta = \omega - \omega_0 \qquad \delta < 0 \qquad |\delta| \gg |\omega_{Rabi}| \qquad \omega \ll \gamma \cdot |\vec{B}_0|$$

Then:

$$\vec{B}_{fict} \neq -|\vec{B}_0|\vec{e}_z$$

Result: We get an effective magnetic field $\vec{B}_{eff}$ with an angle φ relative to the $\vec{z}$-axis.





The series of pictures shows what happens. With increasing angle φ the magnetic field $\vec{B}_{eff}$ rotates from the direction of $\vec{B}$ to $-\vec{B}$.

The precessing magnetic moment $\vec{\mu}$ follows.

If the detuning is increased slowly compared to the Larmor frequency, the spin will continue to precess tightly around $\vec{B}_{eff}$.

For $\delta = 0$ we have $\vec{B}_{eff}$ pointing along the x-axis, for $\delta \gg \omega_{Rabi}$ along the $-\vec{e}_z$ axis.

We calculate $tan\ \varphi$:

$$\tan\varphi = \frac{\left|\overrightarrow{B_1}\right|}{\left|\overrightarrow{B_0}\right| - \frac{\omega}{\gamma}} = \frac{\omega_{Rabi}}{\omega_0 - \omega} = -\frac{\omega_{Rabi}}{\delta}$$

The magnetic moment, starting out along $\overrightarrow{B_0} = \left|\overrightarrow{B_0}\right| \cdot \vec{e}_z$, ends up pointing along $-\overrightarrow{B_0} = \left|\overrightarrow{B_0}\right| \cdot \vec{e}_z$.

Note that in the rotating frame μ remains always (almost) parallel to the effective field $\vec{B}_{eff}$.

As the electron moves in this field, the fast precession of the magnetic moment about the local field keeps its direction locked to the local field.

When B_{eff} points along the $\vec{e}_x$-axis, the generalized Rabi frequency is smallest and equal to the resonant Rabi frequency ω_{Rabi} at $\delta = 0$.

This is a “critical point”. If $|\dot{\varphi}|$ is too big there is a chance the precession of the electron being caught by the static field $\overrightarrow{B_1}$

At this point the adiabatic requirement is most severe there, near $\theta = \frac{\pi}{2}$.

There we have, with $\left|\overrightarrow{B_z}\right| = \left|\overrightarrow{B_0}\right| - \frac{\omega(t)}{\gamma}$:

$$|\dot{\varphi}| = \frac{\left|\dot{\vec{B}}_z\right|}{\left|\overrightarrow{B_1}\right|} = \frac{|\dot{\omega}|}{\gamma \cdot \left|\overrightarrow{B_1}\right|} = \frac{|\dot{\omega}|}{\omega_{Rabi}} \overset{!}{\ll} \omega_{Rabi}$$

The exclamation mark above $\ll$ indicates the requirement which we impose.

It follows that if we want to have this process to be adiabatic, we must adjust $|\dot{\omega}| \ll {\omega^2}_{Rabi}$.

With this we calculate the change $\Delta\omega$ of rotation frequency ω per Rabi period $T = \frac{2\pi}{\omega_{Rabi}}$:

$$|\Delta\omega| = |\dot{\omega}| \cdot T = \frac{|\dot{\omega}|}{2\pi \cdot \omega_{Rabi}}$$

$|\Delta\omega|$ must be small compared to the Rabi frequency ω_{Rabi}.

Note: The inversion of the spin is independent of whether ω is swept up or down through resonance.

Note: If we have several electrons in the magnetic field we will flip all electron spins together.