The infinite potential well is paradigmatic for quantum mechanics. Some solutions work with trigonometric functions only, others use exponential functions.

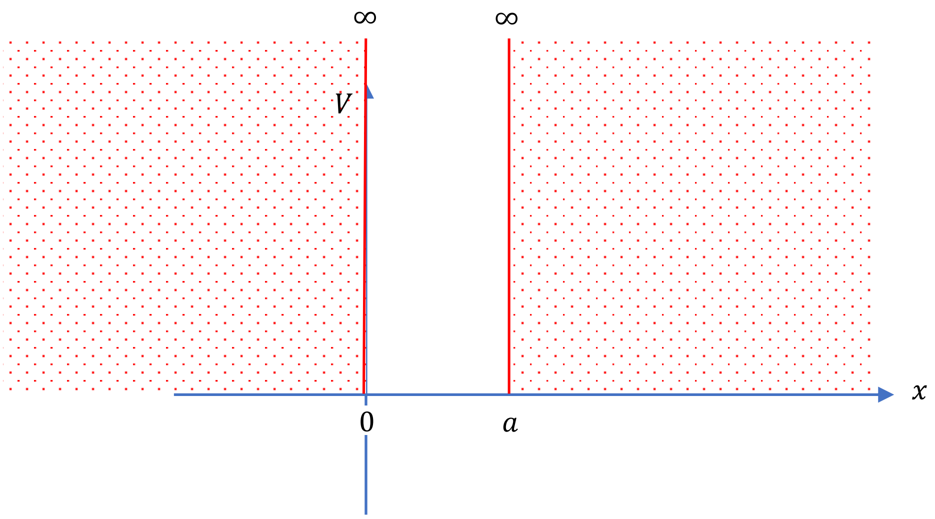
This paper works through the infinite potential well parallel with exponential and trigonometric approach and shows that both ways lead to the same result.

In Griffiths you find information in Chapter 2.6 The Finite Square Well.

An application of the quantum well dealing with led and solar cells you may find at

http://www-personal.umich.edu/~kubarych/journal_pdfs/j_chem_571_1_11_2005.pdf

Hope I can help you with learning quantum mechanics.



The Schrödinger equation for the infinite potential well in one dimension:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

In the well the potential is zero:

$$\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + E\psi(x) = 0$$

This is a differential equation, written as:

$$\frac{d^2\psi(x)}{dx^2} + k^2\psi(x) = 0$$

$$k^2 = \frac{2mE}{\hbar^2}$$

Boundary condition: the wave function must vanish at $x = 0$ and $x = a$.

Solution with exponential functions	Solution with trigonometric functions
$\psi_1(x) = Ae^{ikx}$ $\psi_2(x) = Be^{-ikx}$ Combined solution $\psi(x) = Ae^{ikx} + Be^{-ikx}$	$\psi_1(x) = \mathcal{A} \cdot sin(kx)$ $\psi_2(x) = \mathcal{B} \cdot cos(kx)$ Combined solution: $\psi(x) = \mathcal{A} \cdot sin(kx) + \mathcal{B} \cdot cos(kx)$
$\psi(0) = 0$ $0 = Ae^{ik0} + Be^{-ik0}$ $0 = A + B$ $B = -A$ Intermediate result: $\psi(x) = A\left(e^{ikx} - e^{-ikx}\right)$	$\psi(0) = 0$ $0 = \mathcal{A} \cdot sin(0) + \mathcal{B} \cdot cos(0)$ $\sin(0) = 0, \cos(0) = 1 \rightarrow B = 0$ Intermediate result: $\psi(x) = \mathcal{A} \cdot sin(kx)$
$\psi(a) = 0$ $0 = A\left(e^{ika} - e^{-ika}\right)$ $e^{ika} - e^{-ika} = 0$ $e^{ika} = e^{-ika}$ $e^{2ika} = 1$ $e^{2ika} = e^{i2n\pi}$ Periodicity of exponential function: $e^{i(2ka-2n\pi)} = 1$ $i(2ka - 2n\pi) = 0$ $ka - n\pi = 0$ $k = \frac{n\pi}{a}$	$\psi(a) = 0$ $0 = \mathcal{A} \cdot sin(ka)$ Periodicity of sin: $\sin(ka) = 0 \rightarrow ka = n\pi$ $k = \frac{n\pi}{a}$
Normalization constraint: $\int_0^a \lvert\psi(x)\rvert^2 dx = 1$ $\int_0^a \psi^*(x)\psi(x) dx = 1$ $\int_0^a A\left(e^{-ikx} - e^{ikx}\right)A\left(e^{ikx} - e^{-ikx}\right)dx =$ $A^2\int_0^a \left(e^{-ikx} - e^{ikx}\right)\left(e^{ikx} - e^{-ikx}\right)dx =$	Normalization constraint: $\int_0^a \lvert\psi(x)\rvert^2 dx = 1$ $\int_0^a \lvert\mathcal{A} \cdot sin(kx)\rvert^2 dx =$ $\mathcal{A}^2\int_0^a sin^2(kx) dx =$ $\mathcal{A}^2 \left\lvert\frac{x}{2} - \frac{1}{4k}\sin(2kx)\right\rvert_0^a =$

$A^2\int_0^a 1-e^{-2ikx}-e^{2ikx}+1dx =$ $A^2\int_0^a 2-\left(e^{-2ikx}+e^{2ikx}\right)dx =$ $2aA^2 - A^2\int_0^a \left(e^{-2ikx}+e^{2ikx}\right)dx\,;$ $\int_0^a \left(e^{-2ikx}+e^{2ikx}\right)dx =$ $\left\|\frac{e^{-2ikx}}{-2ik}\right\|_0^a + \left\|\frac{e^{2ikx}}{2ik}\right\|_0^a =$ $\frac{e^{-2ika}-1}{-2ik}+\frac{e^{2ika}-1}{2ik}$ $\frac{1-e^{-2ika}+e^{2ika}-1}{2ik} =;$ Note: $k=\frac{n\pi}{a}$ $\frac{-e^{-2in\pi}+e^{2in\pi}}{2ik}=0$ Normalization constraint: $2aA^2=1 \to A=\sqrt{\frac{1}{2a}}$	$\mathcal{A}^2\left(\frac{a}{2}-\frac{1}{4k}\sin(2ka)\right)=;$ Note: $k=\frac{n\pi}{a}$ $\mathcal{A}^2\left(\frac{a}{2}-\frac{1}{4k}\sin(2ka)\right)=$ $\mathcal{A}^2\left(\frac{a}{2}-\frac{a}{4n\pi}\sin(2n\pi)\right)=$ $\mathcal{A}^2\frac{a}{2}=1$ $\mathcal{A}=\sqrt{\frac{2}{a}}$
We have different results for A and $\mathcal{A}$: $A=\sqrt{\frac{1}{a}}\cdot\sqrt{\frac{1}{2}}$	$\mathcal{A}=\sqrt{\frac{1}{a}}\cdot\sqrt{2}$
The wave functions differ too: $\psi(x)=A\left(e^{ikx}-e^{-ikx}\right)$ We examine: $e^{ikx}-e^{-ikx}=2i\cdot sin(kx)=$ $2\cdot sin(kx)\cdot e^{i\frac{\pi}{2}}$ We ignore the global phase $e^{i\frac{\pi}{2}}$ and get: $\psi(x)=2A\cdot sin(kx)=$ $2\cdot\sqrt{\frac{1}{a}}\cdot\sqrt{\frac{1}{2}}\cdot sin(kx)=$ $\sqrt{\frac{1}{a}}\cdot\sqrt{2}\cdot sin(kx)$	$\psi(x)=\mathcal{A}\cdot sin(kx)$
Both wave functions are identic except for a global phase factor that becomes explicit in the exponential access.	