

We can construct an uncertainty relation out of the classical damped harmonic oscillator.

Hope I can help you with learning quantum mechanics.

Classically, a resonance is a system where one or more variables change periodically such that when the system is no longer driven by an extended periodic process, the decay of the system's oscillation takes many or at least several oscillation periods.

Equivalently, the response of the driven system as function of drive frequency exhibits some form of peaked structure.

The frequency corresponding to the maximum response of the system is called the resonance frequency ( $f_0$ ).

More information you may find at <https://scholar.harvard.edu/files/schwartz/files/lecture2-driven-oscillators.pdf>

We use:

The differential equation for a damped driven oscillator:

$$\frac{d^2}{dt^2} + \gamma \cdot \frac{dx}{dt} + \omega_0^2 \cdot x = F(t)$$

We work with the homogeneous differential equation:

$$\frac{d^2}{dt^2} + \gamma \cdot \frac{dx}{dt} + \omega_0^2 \cdot x = 0$$

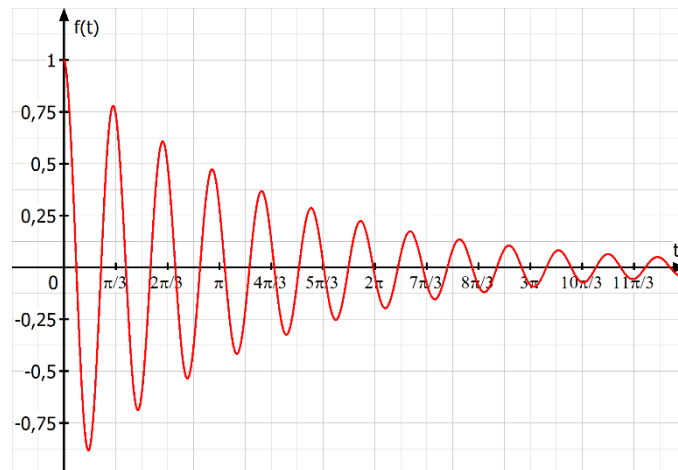
We get a solution:

$$x(t) = A \cdot e^{-\frac{\gamma}{2}t} \cos(\omega_0 t + \phi)$$

We use damping factor  $\gamma = \frac{1}{2}$ ,  $\omega_0 = \frac{2\pi}{T}$ ,  $T = 1$ ,  $A = 1$ ,  $\phi = 0$ .

We get:

$$x(t) = e^{-\frac{1}{4}t} \cos(2\pi t)$$



Now we look at a driven oscillator:

$$\frac{d^2}{dt^2} + \gamma \cdot \frac{dx}{dt} + \omega_0^2 \cdot x = F(t)$$

$$F(t) = \frac{F_0}{m} \cdot \cos(\omega_d t)$$

We use:

$$F_0 = 1, m = 2, T = 1$$

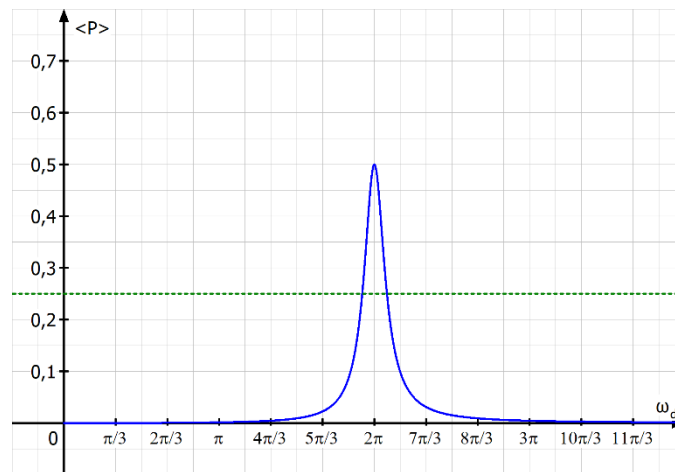
The average power  $\langle P \rangle$  put into the system over a period of  $T = \frac{2\pi}{\omega_d}$ :

$$\langle P \rangle = \left( \frac{F_0^2}{2\gamma m} \right) \frac{(\gamma \omega_d)^2}{(\omega_0^2 - \omega_d^2)^2 + (\gamma \omega_d)^2}$$

Note:  $0 \leq \langle P \rangle \leq 1$ .

Using our parameters we get:

$$\langle P \rangle = \frac{1}{2} \cdot \frac{\left(\frac{1}{2}\omega_d\right)^2}{(4\pi^2 - \omega_d^2)^2 + \left(\frac{1}{2}\omega_d\right)^2} = \frac{1}{8} \cdot \frac{\omega_d^2}{(4\pi^2 - \omega_d^2)^2 + \left(\frac{1}{2}\omega_d\right)^2}$$



The dashed line is half of the maximum power transfer located at  $\omega_0$ .

The width at half-maximum is  $\gamma$ .

The curve is called a Lorentzian.

If we now use that  $\gamma$  is a kind of frequency uncertainty,  $\Delta\omega_d$  and set the damping time  $\tau$  so that the power transfer has dropped to  $\frac{1}{e}$  we roughly get:

$$\Delta\omega_d \cdot \tau \approx 1$$

We multiply with  $\hbar$ :

$$\Delta\hbar\omega_d \cdot \tau \approx \hbar$$

Using  $\Delta\hbar\omega_d = \Delta E$  we get:

$$\Delta E \cdot \tau \approx \hbar$$

This resembles the uncertainty relation.